Physics of the Dark Universe 19 (2018) 129–136

Contents lists available at ScienceDirect

Physics of the Dark Universe

journal homepage: www.elsevier.com/locate/dark

Non-Abelian S-term dark energy and inflation

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ARTICLE INFO

Article history:
Received 27 November 2017
Received in revised form 22 January 2018
Accepted 24 January 2018

Keywords:
Vector fields
Dark energy
Inflation

ABSTRACT

We study the role that a cosmic triad in the generalized $SU(2)$ Proca theory, specifically in one of the pieces of the Lagrangian that involves the symmetric version $S_{\mu\nu}$ of the gauge field strength tensor $F_{\mu\nu}$, has on dark energy and primordial inflation. Regarding dark energy, the triad behaves asymptotically as a couple of radiation perfect fluids whose energy densities are negative for the S term but positive for the Yang–Mills term. This leads to an interesting dynamical fine-tuning mechanism that gives rise to a combined equation of state parameter $\omega \simeq -1$ and, therefore, to an eternal period of accelerated isotropic expansion for an ample spectrum of initial conditions. Regarding primordial inflation, one of the critical points of the associated dynamical system can describe a prolonged period of isotropic slow-roll inflation sustained by the S term. This period ends up when the Yang–Mills term dominates the energy density leading to the radiation dominated epoch. Unfortunately, in contrast to the dark energy case, the primordial inflation scenario is strongly sensitive to the coupling constants and initial conditions. The whole model, including the other pieces of the Lagrangian that involve $S_{\mu\nu}$, might evade the recent strong constraints coming from the gravitational wave signal GW170817 and its electromagnetic counterpart GRB 170817A.

1. Introduction

The vector sector of gauge field theories is built from the gauge field strength tensor $F_{\mu\nu}$, its Hodge dual $\tilde{F}_{\mu\nu}$, and, if the gauge symmetry is spontaneously broken, from the vector field A_μ [1]. Generalized Proca theories have taught us that, when the gauge symmetry is explicitly broken, the vector sector of these theories is also built from the symmetric version $S_{\mu\nu}$ of $F_{\mu\nu}$ [2,3] (see also Refs. [4,5]). The cosmological implications of $F_{\mu\nu}$, $\tilde{F}_{\mu\nu}$, and A_μ have been well investigated in the literature (see, for instance Refs. [6–8]) but little has been said about $S_{\mu\nu}$. In this paper, we study the cosmological implications of a cosmic triad [9] in the vector–tensor Horndeski theory, also called the theory of vector Galileons, endowed with a global $SU(2)$ symmetry. In particular, we analyse the Yang–Mills Lagrangian together with $\mathcal{L}_4^1 \subset \mathcal{L}_4$, it being one of the pieces of the generalized $SU(2)$ Proca Lagrangian [10] that contains contractions of two $S_{\mu\nu}$. We have found an asymptotic behaviour in which the cosmic triad under $\mathcal{L}_4^1$ behaves as an almost radiation-like perfect fluid with negative energy density and pressure whose absolute values matches almost precisely those of the radiation perfect fluid coming from the same cosmic triad under the Yang–Mills Lagrangian. The system exhibits an interesting dynamical fine-tuning mechanism which results in a combined equation of state parameter $\omega \simeq -1$ and, therefore, in an eternal isotropic inflationary period; this makes of this model an ideal candidate to explain the dark energy. We have also explored the dynamical system associated to this model and we have found that one of the critical points may correspond to a prolonged period of isotropic slow-roll accelerated expansion. This is a saddle point, i.e., it represents a transient state of the dynamical system so that the inflationary period comes naturally to an end, this being replaced by a radiation dominated period by virtue of the Yang–Mills Lagrangian; this model would be an ideal candidate to explain the primordial inflation were it not for the necessary judicious choosing of initial conditions and parameters in the action. The purpose of this paper is to isolate and understand the cosmological implications of $\mathcal{L}_4^1$ despite of being apparently strongly constrained [11–15] by the recent observation of the gravitational wave signal GW170817 [16] and its electromagnetic counterpart GRB 170817A [17,18].[1] The purpose is reasonable since the generalized $SU(2)$ Proca Lagrangian contains $\mathcal{L}_4 \equiv \alpha\mathcal{L}_4^1 + \kappa\mathcal{L}_4^2 + \lambda\mathcal{L}_4^3$,

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[1] We say "apparently strongly constrained" because there does not exist a formal proof of it. The analyses so far done are for a scalar Galileon [12–15] and for the generalized Proca action for an Abelian vector field [11].

https://doi.org/10.1016/j.dark.2018.01.003
2212-6864/

α, κ, λ being constants, where a relation between α and κ might be established so that the gravitational waves speed matches that of light.[2] In such a scenario, although $\alpha, \kappa \neq 0$ in principle, κ being a function of α, it might happen the cosmological implications of $\mathcal{L}_4^1$ are not counterbalanced by those of $\mathcal{L}_4^2$. We will analyse such a scenario and the whole cosmological implications of $\alpha\mathcal{L}_4^1 + \kappa\mathcal{L}_4^2 \subset \mathcal{L}_4$ in a forthcoming publication.

2. Generalized Proca theories and the cosmic triad

Generalized Proca theories are built following the same construction idea of the Galileon–Horndeski theories [4,19]. Whatever choices Nature had to define the action, once the field content and the symmetries were decided, all of them must comply with a Hamiltonian bounded from below. And this may be possible, according to Ostrogradski [20], if the dynamical field equations are, at most, second order in space–time derivatives. If the latter condition were not satisfied, the system would generically enter in a severe instability, called Ostrogradski's, both at the classical and quantum levels [21,22]. The traditional approach to construct such theories is by employing scalar fields as the field content [23–28]. Nothing significantly new, compared to the usual canonical kinetic term, is obtained when employing, instead, an Abelian gauge field [29,30]. Hence, having new phenomenology requires no longer invoking gauge symmetries, i.e., it requires a generalization to the Proca action. Such a generalization was performed in Refs. [2,3,31–34] where it was recognized that, besides $F_{\mu\nu}$ and its Hodge dual $\tilde{F}_{\mu\nu}$, the action is also defined in terms of A_μ and the symmetric version $S_{\mu\nu}$ of $F_{\mu\nu}$: $S_{\mu\nu} \equiv \nabla_\mu A_\nu + \nabla_\nu A_\mu$. The application of all these ideas to non-Abelian theories culminated in the construction of the generalized $SU(2)$ Proca theory [10] (see also Ref. [35]). An interesting aspect of this theory is the explicit violation of the $SU(2)$ gauge symmetry which allows a mass term and its generalizations written in terms of the non-Abelian versions of A_μ, $F_{\mu\nu}$, $\tilde{F}_{\mu\nu}$, and $S_{\mu\nu}$. Another interesting aspect is the global character of the $SU(2)$ symmetry which might play an important role in particle physics.[3] A third interesting aspect is the possibility of using a cosmic triad [9], a set of three vector fields mutually orthogonal and of the same norm, which corresponds to an invariant configuration both under $SU(2)$, for the field space, and $SO(3)$, for the physical space, in agreement with the local homomorphism between these two groups. The cosmic triad configuration has been employed before [36–42] and, at least in the Gauge-flation scenario [36,37], its naturalness has been shown in the sense that it is an attractor in a more general anisotropic setup [43]. The cosmological implications of the generalized Proca theory for an Abelian vector field have been recently studied [32,44–47] but always working with a time-like vector field so that the spatial components are chosen to vanish, avoiding this way disastrous anisotropies.[4] In contrast, the isotropic configuration provided by the cosmic triad, although the latter is composed of vector fields that inherently define privileged directions, is amply favoured by cosmological observations. It is the purpose of this paper to focus on the spatial components of a triad of space-like vector fields.

3. The non-Abelian *S* terms and the considered model

The Lagrangian of the generalized $SU(2)$ Proca theory is composed of several pieces that are described in Eqs. (96)–(99) of Ref. [10]. Of particular importance is $\mathcal{L}_4$ which is characterized by the two first-order covariant space–time derivatives of A_μ that each of its terms contain (except for the non-minimal coupling to gravity terms):

$$\mathcal{L}_4 \equiv \alpha\mathcal{L}_4^1 + \kappa\mathcal{L}_4^2 + \lambda\mathcal{L}_4^3 \,, \tag{1}$$

with $\alpha, \kappa, \lambda \in \mathbb{R}$ and where[5]

$$\begin{aligned}\mathcal{L}_4^1 \equiv\ & \frac{1}{4}(A_b \cdot A^b)\left[S_\mu^{\mu a} S_{\nu a}^{\nu} - S_\nu^{\mu a} S_{\mu a}^{\nu} + A_a \cdot A^a R\right] \\ & + \frac{1}{2}(A_a \cdot A_b)\left[S_\mu^{\mu a} S_\nu^{\nu b} - S_\nu^{\mu a} S_\mu^{\nu b} + 2A^a \cdot A^b R\right] \,, \end{aligned} \tag{2}$$

$$\begin{aligned}\mathcal{L}_4^2 \equiv\ & \frac{1}{4}(A_a \cdot A_b)\left[S_\mu^{\mu a} S_\nu^{\nu b} - S_\nu^{\mu a} S_\mu^{\nu b} + A^a \cdot A^b R\right] \\ & + \frac{1}{2}(A^{\mu a} A^{\nu b})\Big[S_{\mu a}^{\rho} S_{\nu\rho b} - S_{\nu a}^{\rho} S_{\mu\rho b} - A_a^\rho A_b^\sigma R_{\mu\nu\rho\sigma} \\ & - \left(\nabla^\rho A_{\mu a}\right)\left(\nabla_\rho A_{\nu b}\right) + \left(\nabla^\rho A_{\nu a}\right)\left(\nabla_\rho A_{\mu b}\right)\Big] \,, \end{aligned} \tag{3}$$

$$\mathcal{L}_4^3 \equiv \tilde{G}_{\mu\sigma}^b A_a^\mu A_{\nu b} S^{\nu\sigma a} \,. \tag{4}$$

In the previous expressions, gauge indices run from 1 to 3 and are represented by Latin letters, space–time indices run from 0 to 3 and are represented by Greek letters, R is the Ricci scalar, $R_{\mu\nu\rho\sigma}$ is the Riemann tensor, $G_{\mu\nu}^a$ is the Abelian version of $F_{\mu\nu}^a$:

$$G_{\mu\nu}^a \equiv \nabla_\mu A_\nu^a - \nabla_\nu A_\mu^a \,, \tag{5}$$

$\tilde{G}_{\mu\nu}^a$ is the Hodge dual of $G_{\mu\nu}^a$, and $S_{\mu\nu}^a$ is the symmetric version of $G_{\mu\nu}^a$:

$$S_{\mu\nu}^a \equiv \nabla_\mu A_\nu^a + \nabla_\nu A_\mu^a \,. \tag{6}$$

It is very important to notice that the third line of $\mathcal{L}_4^2$, formed by products of two first-order covariant space–time derivatives of A_μ, cannot be written either in terms of $F_{\mu\nu}^a$, $\tilde{F}_{\mu\nu}^a$, or $S_{\mu\nu}^a$, this line being a specific term to the non-Abelian nature of the theory [10]. As such, it vanishes in the Abelian case so that $\mathcal{L}_4^1 + \mathcal{L}_4^2$ reduces to $-A^2[(S_\mu^\mu)^2 - S_\rho^\sigma S_\sigma^\rho] + \frac{1}{4}A^4 R$ which is part of the corresponding $\mathcal{L}_4$ in the generalized Proca theory for an Abelian vector field [4]. This is the reason why we will denote $\mathcal{L}_4^1$ and $\mathcal{L}_4^2$ as the non-Abelian *S* terms. In this paper, we will analyse the cosmological consequences of the non-Abelian *S* term in the action

$$\mathcal{S} = \int d^4x \sqrt{-\det(g_{\mu\nu})}\,(\mathcal{L}_{\text{E-H}} + \mathcal{L}_{\text{YM}} + \alpha\mathcal{L}_4^1) \,, \tag{7}$$

where $g_{\mu\nu}$ is the metric tensor, $\mathcal{L}_{\text{E-H}}$ is the Einstein–Hilbert Lagrangian,

$$\mathcal{L}_{\text{YM}} \equiv -\frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu} \,, \tag{8}$$

is the canonical kinetic term of A_μ, and

$$F_{\mu\nu}^a \equiv \nabla_\mu A_\nu^a - \nabla_\nu A_\mu^a + g\epsilon^a{}_{bc} A_\mu^b A_\nu^c \,, \tag{9}$$

where g is the coupling constant of the group whereas the group structure constants are given by the Levi-Civita symbol ϵ_{abc}.

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[2] The cosmological implications of $\mathcal{L}_4^3$ were reported in Ref. [4]. For its own existence, this parity-violating term requires not only at least one non-vanishing temporal component but also a non-orthogonal configuration for the triad, potentially generating anisotropies in the expansion in conflict with observations.

[3] Global continuous symmetries are important in particle physics, say, for example, in the solution to the strong CP problem via the spontaneous breaking of the $U(1)$ global symmetry imposed by the Peccei–Quinn mechanism [1].

[4] An exception is the model studied in Ref. [48] where a triad of space-like Abelian vector fields is considered so that the temporal components are chosen to vanish. The results of this work are very interesting despite the unnaturalness of the triad configuration when there is no an underlying global $SU(2)$ symmetry.

[5] The difference between our $\mathcal{L}_4^1$ and that in Ref. [10] is $[(A_b \cdot A^b)G_\nu^{\mu a} G_{\mu a}{}^\nu + 2(A_a \cdot A_b)G_\nu^{\mu a} G_\mu^{b\,\nu}]/4$. Likewise, the difference between our $\mathcal{L}_4^2$ and that in Ref. [10] is $[(A_a \cdot A_b)G_\nu^{\mu a} G_\mu^{b\,\nu} - 2(A^{\mu a}A^{\nu b})(G_{\mu a}^{\ \ \rho} G_{\nu\rho b} - G_{\nu a}^{\ \ \rho} G_{\mu\rho b})]/4$. These differences formally belong to $\mathcal{L}_2 \equiv \mathcal{L}_2(A_\mu^a, G_{\mu\nu}^a, \tilde{G}_{\mu\nu}^a)$ in Eq. (96) of Ref. [10].