

Electric charge quantization in $SU(3)_c \otimes SU(4)_L \otimes U(1)_X$ models

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Abstract

We show how in a model based on the gauge group $SU(3)_c \otimes SU(4)_L \otimes U(1)_X$, the quantization of the electric charge can be explained.

Keywords: Grand unification theory, electric charge quantization

One of the most intriguing questions is about why the electric charge is quantized. It remains as an open question although there are some proposals like the first one given by Dirac proposing a quantum mechanical theory including magnetic monopoles which implies that electric charge should be quantized [1]. The second idea is coming from grand unification theory using the group structure itself. There is another one, in gauge models that contain explicitly the $U(1)$ abelian group in its structure and contributes to the $U(1)_{em}$ after the spontaneous symmetry breaking [2]. Using this last proposal some studies have been done in the framework of the $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ (331) models [7] [8] where using classical and quantum constraints some relationships between the $U(1)$ charges are obtained and they lead to electric charge quantization. Here it is interesting that in the standard model with one family the electric charge quantization can be obtained but considering the three families and neutrinos without mass then a dequantization arises up. It is possible to restore the electric charge quantization if Majorana neutrinos are included and all this is related with the global hidden symmetry $U(1)_{B-L}$. On the other hand, in the framework of the 331 models the electric charge quantization is achieved when three families are involved together and it does not depend of the neutrinos, massless or not.

The model 331 which enlarges the gauge group of the standard model share with the $SU(3)_c \otimes SU(4)_L \otimes U(1)_X$ (341) version the interesting feature of addressing the

problem of the number of fermion families. It is concern to the anomaly cancellation among the families which is obtained if the number of left-handed triplets is equals to the antitriplets in the 331 model and equal number of 4-plets and 4^* -plets in the 341 model, taking into account the color degree of freedom. On the other hand, if we accept a right handed neutrino, one option is to have ν , e^- , ν^c and e^c in the same multiplet of $SU(4)_L$. Then using the lightest leptons as the particles which determine the gauge symmetry, if each generator is treated separately, the gauge group $SU(4)$ is the highest symmetry group to be considered in the electroweak sector. Models based on the gauge symmetry $SU(3)_c \otimes SU(4)_L \otimes U(1)_N$ have been studied before [3] and also this symmetry appears in some little Higgs models [4]. In this work we use a model based on $SU(3)_c \otimes SU(4)_L \otimes U(1)_N$ gauge group presented by [5] in order to show how the electric charge quantization is satisfied, we mention that this work is based on the results reported by authors in reference [6].

We are going to consider a model based on the gauge group $SU(3)_c \otimes SU(4)_L \otimes U(1)_X$ already built up by Pisano and Pleitez [5]. The electric charge operator is defined as a linear combination of the diagonal generators of the group

$$Q = \frac{1}{2} \left(\lambda_3 - \frac{\lambda_8}{\sqrt{3}} - \frac{2\sqrt{6}}{3} \lambda_{15} \right) + bX \quad (1)$$

where λ_i are the Gell-Mann matrices for $SU(4)$ and X

the quantum number associated to $U(1)_X$. The quark spectra of the model under consideration is composed by

$$q_{\alpha L}^T = (j_\alpha, d'_\alpha, u_\alpha, d_\alpha)_{(3,4^*,X_{q_\alpha}^L)}, \quad (2)$$

$$q_{1L}^T = (u_1, d_1, u', J)_{(3,4,X_{q_1}^L)}, \quad (3)$$

$$f_{iL}^T = (v_i, \ell_i, \nu_i^c, \ell_i^c)_{(1,4,X_{f_i}^L)} \quad (4)$$

$$u_{\alpha R} \sim (3, 1, X_u^R), \quad d_{\alpha R} \sim (3, 1, X_d^R), \quad (5)$$

$$u'_R \sim (3, 1, X_{u'}^R), \quad d'_R \sim (3, 1, X_{d'}^R) \quad (6)$$

$$j_{\alpha R} \sim (3, 1, X_{j_\alpha}^R), \quad J \sim (3, 1, X_J^R), \quad (7)$$

$$\ell_{iR} \sim (1, 1, X_{\ell_i}^R) \quad (8)$$

where $\alpha = 1, 2$ and their assigned quantum numbers of $SU(3)_C \otimes SU(4)_L \otimes U(1)_X$ are shown in the parenthesis.

On the other hand, the scalar sector needed in order to get quark masses is

$$\eta^T = (\eta_1^0, \eta_1^-, \eta_2^0, \eta_2^+)_{(1,4,X_\eta)}, \quad (9)$$

$$\rho^T = (\rho_1^+, \rho^0, \rho_2^+, \rho^{++})_{(1,4,X_\rho)}, \quad (10)$$

$$\chi^T = (\chi_1^+, \chi^{--}, \chi_2^-, \chi^0)_{(1,4,X_\chi)} \quad (11)$$

and it is necessary to introduce a Higgs boson in a 10 representation of the gauge group $SU(4)$ in order to give mass to the leptonic sector,

$$H = \begin{pmatrix} H^0 & H_1^+ & H_2^0 & H_3^- \\ H_1^+ & H_1^{++} & H_3^+ & H_3^0 \\ H_2^0 & H_2^+ & H_4^0 & H_4^- \\ H_3^- & H_3^0 & H_4^- & H_2^- \end{pmatrix}. \quad (12)$$

Finally, the Yukawa Lagrangian is

$$\begin{aligned} -\mathcal{L}_Y &= \frac{1}{2} G_{ij} \overline{f_{iL}} f_{jL} H + F_{1k} \overline{q_{1L}} u_{kR} \\ &+ F_{\alpha k} \overline{q_{\alpha L}} u_{kR} \rho^* + F'_{1k} \overline{q_{1L}} d_{kR} \rho \\ &+ F_{\alpha\beta} \overline{q_{\alpha L}} d_{\beta R} \eta^* + h_1 \overline{q_{1L}} u'_{R} \eta' \\ &+ h_{\alpha\beta} \overline{q_{\alpha L}} d'_{\beta R} \eta'^* + \Gamma_1 \overline{q_{1L}} J_{R\chi} \\ &+ \Gamma_{\alpha\beta} \overline{q_{\alpha L}} j_{\beta R} \chi^* + \text{h.c.} \end{aligned} \quad (13)$$

where the field η' is introduced in the same way as the η , and has the same quantum numbers as η but it has the following vacuum expectation value $\langle \eta' \rangle = (0, 0, v_0, 0)$.

Now we are going to consider some theoretical constraints that will help us achieve the electric charge quantization in the 341 model. First of all, the electric charge operator should annihilate the vacuum in order

to have electric charge conservation. With that consideration we get

$$\begin{aligned} b &= \frac{1}{X_\rho}, & X_\eta &= 0, \\ X_\chi &= -X_\rho & X_H &= 0. \end{aligned} \quad (14)$$

Now, we consider the restrictions using that the Yukawa Lagrangian must be invariant under $U(1)$ transformations. They are

$$X_{f_1} = X_{f_2} = X_{f_3} = 0, \quad (15)$$

$$X_{q_1}^L = X_{u_k}^R = X_{u'}^R \quad (16)$$

$$X_{q_\alpha}^L = X_{d_k}^R = X_{d'}^R \quad (17)$$

$$X_{u_k}^R = X_{q_\alpha}^L + X_\rho \quad (18)$$

$$X_{q_1}^L = X_{d_k}^R + X_\rho \quad (19)$$

$$X_J^R = X_{q_1}^L + X_\rho \quad (20)$$

$$X_{q_\alpha}^L = X_{j_\beta}^R + X_\rho. \quad (21)$$

Additionally, the quantum constraints that lead to the vanishing of the quiral anomaly coefficients imply the following relations

$$[SU(3)_C]^2 \otimes U(1)_X \rightarrow 3 \sum_m X_{q^{(m)}}^L - \sum_{\text{sing}} X_q^R \quad (22)$$

$$[SU(4)_L]^2 \otimes U(1)_X \rightarrow \sum_m X_{f^{(m)}}^L + 3 \sum_m X_{q^{(m)}}^L \quad (23)$$

Therefore, taking into account the anomaly conditions, we arrive to

$$X_{q_1}^L + 2X_{q_\alpha}^L = 0 \quad (24)$$

where we have used the anomaly coming from $[SU(4)_L]^2 \otimes U(1)_X$ and equation (15). Furthermore, considering (24) and (16), we obtain

$$X_{q_1}^L = \frac{2}{3} X_\rho, \quad X_{q_\alpha}^L = -\frac{1}{3} X_\rho \quad (25)$$

$$X_{u_k}^R = \frac{2}{3} X_\rho, \quad X_{d_k}^R = -\frac{1}{3} X_\rho \quad (26)$$

$$X_{u'}^R = \frac{2}{3} X_\rho, \quad X_{d'}^R = -\frac{1}{3} X_\rho \quad (27)$$

$$X_J^R = \frac{5}{3} X_\rho, \quad X_{j_\beta}^R = -\frac{4}{3} X_\rho \quad (28)$$

$$(29)$$

and using the above equations, it is possible to re-write the electric charge operator in this model as

$$Q = \text{Diag}(0, -1, 0, 1) + \frac{X_\phi}{X_\rho} I_{4 \times 4} \quad (30)$$

In summary, we have described that electric charge can be quantized in models based on the gauge symmetry $SU(3)_c \otimes SU(4)_L \otimes U(1)_X$ that use three families to cancel out the chiral anomalies. We have shown this quantization using electric charge conservation, invariance under parity transformations and chiral anomaly conditions.

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