

CHARGED LEPTON MIXING PROCESSES IN 331 MODELS

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Processes $\tau \rightarrow l\gamma$, $\tau \rightarrow ll$ with $l = e, \mu$ and $\mu(\tau) \rightarrow e(\mu)\gamma$ are evaluated in the framework of a model based on the extended symmetry gauge $SU(3)_c \otimes SU(3)_L \otimes U(1)_Y$ with a leptonic sector consisting of five triplets. Lepton flavor violating processes are allowed at tree level in this model through the new Z' gauge boson. We obtained bounds for the mixing angles in the leptonic sector of the model, considering the experimental measurements of the processes from the BELLE and the BABAR collaborations.

Keywords: 331 models; beyond the standard model; FCNC; Z' boson; lepton decays.

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1. Introduction

In the framework of the standard model (SM) of high energy physics, there are many unclear issues that require extensions of the theory in the local symmetry and in the particle spectrum. One of these issues is the flavor puzzle: why there are three families of fermions and why they have their peculiar structure of masses and mixings. There are many studies of these problems in the quark sector where there are different observables in the up and down sector. Usually, the mixing and masses pattern are studied through the flavor changing neutral current (FCNC) processes and constraints on flavor symmetries are obtained.¹⁻¹² In the leptonic sector, many analysis have been done in the neutrino sector using specially the available data coming from different neutrino oscillation experiments.¹³⁻¹⁶ An important sector of phenomenology is a concern to charged lepton flavor violating (LFV) processes

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because they provide direct information about the flavor structure of the lepton sector. Phenomenologically, there are different models and extensions of the SM that explain LFV. One common approach is to include nonrenormalizable effective operators of dimension five and six in order to have a source of LFV processes.¹⁷ The effective operators approach is quite general to look for the new physics effects at low energies, but the limits obtained on the LFV couplings are not translated easily to specific models.^{18,19}

Experimentally, there are searches of charged LFV processes using channels like $\mu \rightarrow e\gamma$, $\tau \rightarrow e(\mu)\gamma$, $\mu \rightarrow eee$ and conversion $e - \mu$ nuclei.^{20,21} The experiment MEG, PSI (Switzerland) gives the best limits on $\mu^+ \rightarrow e^+\gamma$, reporting $\text{Br}(\mu^+ \rightarrow e^+\gamma) < 2.4 \times 10^{-12}$ and they expect to reach a sensibility $\approx \text{few} \times 10^{-13}$. Similarly, for the process $\mu^+ \rightarrow e^+e^+e^-$ is reported by SINDRUM I to be in the range of 10^{-12} and the next collaborations MuSIC and $\mu 3e$ expect to reach up to $\approx 10^{-16} - 10^{-15}$. And for the processes $\mu - e$ nuclei conversion, the rate reported by SINDRUM II is $R_{\mu e}(\text{Au}) < 7 \times 10^{-13}$ and the future experiments Mu2e (FNLA) and COMET (J-PARC) are expected to reach $R_{\mu e}(\text{Al}) \approx 10^{-16}$. On the other hand, LFV processes have also been searched in the τ lepton sector and around 48 decay channels have been studied by BABAR and BELLE.²² The best results obtained at 90% C.L. are $\text{Br}(\tau \rightarrow e\gamma) < 3.3 \times 10^{-8}$, $\text{Br}(\tau \rightarrow \mu\gamma) < 4.4 \times 10^{-8}$ and three body decays $\text{Br}(\tau^- \ell_1^- \ell_2^+ \ell_3^-) \lesssim (1.5 - 3.0) \times 10^{-8}$ with $l = e, \mu$.²³⁻²⁵ Future experiments like SuperB or BELLE II could get sensibilities of the order of 10^{-9} . And finally, one new player in the experimental environment is arriving and it is large hadron collider beauty (LHCb), they have reported an upper bound on $\text{Br}(\tau^- \rightarrow \mu^- \mu^- \mu^+) < 6.3 \times 10^{-8}$.²⁶

One of the motivations of physics beyond the SM is to solve the flavor problem and explain the patterns and mixings in the fermion phenomenology. In grand unified theories, different representations are assigned to the fermions and therefore different patterns can emerge in the quark sector, as well as in the lepton sector. LFV processes in the framework of the extended gauge theories is one option to test these models. One possible alternative is based on the gauge symmetry $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$, known as 331 models.²⁷⁻⁴² These models can explain why there are three fermionic families through the chiral anomaly cancellation condition and the number of colors in quantum chromodynamics (QCD). On the other hand, the models based on the 331 symmetry are built in such a way that the couplings of the quarks with the new neutral Z' boson are not universal in the interaction basis, making them not diagonal in the mass eigenstates basis and yielding to FCNC at tree level.³⁻¹⁰ This is a special feature of the 331 models, due to one quark family being in a different representation of the gauge group to the other two families, in order to satisfy the chiral anomaly cancellation condition. It is worth mentioning that in some 331 models there are not only contributions from the left-handed neutral current, but also from the right-handed neutral currents. There are many studies of these new FCNC in the quark sector where are different observables in the up and down sectors that constrain such kind of processes.³⁻¹⁰ In contrast, there are

not so many analysis in the leptonic sector, where leptonic flavor violation (LFV) processes at tree level are present. In particular, LFV processes such as $\tau \rightarrow l^- l^+ l^-$ with $l = e, \mu$, have been discussed in the framework of the minimal supersymmetric standard model, Little Higgs models, left–right symmetry models and many other extensions of the SM.^{43–48} Some of these models predict branching fractions for $\tau \rightarrow l^- l^+ l^-$ of the order of 10^{-7} which could be detected in future experiments.

Different 331 models can be built,^{49–53} they can be distinguished using the electric charge of the new particles introduced in the spectrum and the structure of the scalar sector, where models without exotic charges will be considered. In general, the 331 models are classified depending on how they cancel the chiral anomalies: there are two models that cancel out the anomalies requiring just one family and eight models where the three families are required. In the three family models, there are four models where the leptons are treated identically, two of them treat two quark generations identically and finally, there are two models where all the lepton generations are treated differently.^{49–53} There is one of these 331 models where the leptonic sector is described by five left-handed leptonic triplets in different representations of the $SU(3)_L$ gauge group. Using these five leptonic representations, it is possible to obtain models where the three known leptons coupled to the Z' boson are very different with respect to the new ones. We concentrate on these models in this paper, studying the LFV processes and obtain constraints on the leptonic mixing matrix. In the next section, we are going to present the main features of the model under consideration and then we focus on the LFV processes, namely $\tau \rightarrow l^- l^+ l^-$ with $l = e, \mu$, and $\mu \rightarrow 3e$, $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu(e)\gamma$.

2. The Model 331

The model considered is based on the local gauge symmetry $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (331), where it is common to write the electric charge generator as a linear combination of the diagonal generators of the group:

$$Q = T_3 + \beta T_8 + X, \tag{1}$$

where the parameter β is used to label the particular type of 331 model considered. For constructing the model we choose $\beta = -1/\sqrt{3}$, which corresponds to models where the new fields in the spectra do not have exotic electric charges.

The quark content of this model is described by

$$\begin{aligned} q_{mL} = \begin{pmatrix} u_m \\ d_m \\ B_m \end{pmatrix}_L &\sim (3, 3, 0), & q_{3L} = \begin{pmatrix} d_3 \\ u_3 \\ T_3 \end{pmatrix}_L &\sim (3, 3^*, 1/3), \\ d^c &\sim (3^*, 1, 1/3), & B_m^c &\sim (3^*, 1, 1/3), \\ u^c &\sim (3^*, 1, -2/3), & T^c &\sim (3^*, 1, -2/3), \end{aligned} \tag{2}$$

where $m = 1, 2$ and their assigned quantum numbers of $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ are shown in the parenthesis.

For the leptonic spectrum, we use

$$\begin{aligned} \Psi_{nL} &= \begin{pmatrix} e_n^- \\ \nu_n \\ N_n^0 \end{pmatrix}_L \sim (1, 3^*, -1/3), & \Psi_L &= \begin{pmatrix} \nu_1 \\ e_1^- \\ E_1^- \end{pmatrix}_L \sim (1, 3, -2/3), \\ \Psi_{4L} &= \begin{pmatrix} E_2^- \\ N_3^0 \\ N_4^0 \end{pmatrix}_L \sim (1, 3^*, -1/3), & \Psi_{5L} &= \begin{pmatrix} N_5^0 \\ E_2^+ \\ e_3^+ \end{pmatrix}_L \sim (1, 3^*, 2/3), \\ e_n^c &\sim (1, 1, 1), & e_1^c &\sim (1, 1, 1), & E_1^c &\sim (1, 1, 1), & E_2^c &\sim (1, 1, 1), \end{aligned} \tag{3}$$

with $n = 2, 3$. The five leptonic triplets together with the quark content insures cancellation of chiral anomalies.^{11,12} Furthermore, note that with this proposed assemble for the leptonic sector, there is only one of the triplets that is not written in the adjoint representation of $SU(3)_L$ and it contains one of the standard lepton families of the SM.

On the other hand, in 331 models without exotic charges, the gauge bosons of the $SU(3)_L$ which transform according to the adjoint representation, are given by

$$\mathbf{W}_\mu = W_\mu^a \frac{\lambda^a}{2} = \frac{1}{2} \begin{pmatrix} W_\mu^3 + \frac{1}{\sqrt{3}}W_\mu^8 & \sqrt{2}W_\mu^+ & \sqrt{2}K_{1\mu}^0 \\ \sqrt{2}W_\mu^- & -W_\mu^3 + \frac{1}{\sqrt{3}}W_\mu^8 & \sqrt{2}\bar{K}_{2\mu}^+ \\ \sqrt{2}\bar{K}_{1\mu}^0 & \sqrt{2}K_{2\mu}^- & -\frac{2}{\sqrt{3}}W_\mu^8 \end{pmatrix}, \tag{4}$$

where λ^a are the Gell-Mann matrices for the considered group. The gauge boson field B_μ is associated with the $U(1)_X$ group which is a singlet under $SU(3)_L$ and it does not have electric charge. Once the gauge boson sector is identified, then the bosons of the neutral sector (W^3, W^8, B) are rotated to get the new neutral gauge bosons A, Z and Z' :

$$\begin{pmatrix} A \\ Z \\ Z' \end{pmatrix} = \begin{pmatrix} S_W & -S_W/\sqrt{3} & C_W\sqrt{1-T_W^2/3} \\ C_W & S_W T_W/\sqrt{3} & -S_W\sqrt{1-T_W^2/3} \\ 0 & -\sqrt{1-T_W^2/3} & -T_W/\sqrt{3} \end{pmatrix} \begin{pmatrix} W^3 \\ W^8 \\ B \end{pmatrix}, \tag{5}$$

where θ_W is the Weinberg's angle defined by $T_W = \tan\theta_W = g'/\sqrt{g^2 + g'^2/3}$, with g and g' the coupling constants of the $SU(3)_L$ and $U(1)_X$ groups, respectively ($S_W = \sin\theta_W, C_W = \cos\theta_W$). In this new basis, the photon A_μ is the gauge boson associated to the charge generator Q , while the Z_μ boson can be identified as the usual Z gauge boson of the SM. Scalar states in these models in general can be considered as real fields, therefore the neutral heavy state $\sqrt{2}\text{Im} K$ decouples from the other neutral bosons, becoming an exact mass eigenstate. However, the vector bosons Z, Z' and $\sqrt{2}\text{Re} K$ are in general mix.⁴⁹⁻⁵³ Then, one can rotate to the

mass eigenstate basis, say Z_1, Z_2, Z_3 (where Z_1 is the ordinary gauge boson seen in high energy experiments) through an orthogonal mixing matrix R :

$$\begin{pmatrix} Z \\ Z' \\ \sqrt{2} \operatorname{Re} K \end{pmatrix} = R \begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \end{pmatrix}. \tag{6}$$

For the purpose of this work, we will assume that $\operatorname{Re} K$ does not mix with the Z and Z' bosons.

About the scalar sector, we are going to break the symmetry in such a way that

$$\begin{aligned} SU(3)_c \otimes SU(3)_L \otimes U(1)_X &\rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_X \\ &\rightarrow SU(3)_c \otimes U(1)_Q \end{aligned} \tag{7}$$

and we use the following three scalars $\phi_1(1, 3^*, -1/3)$ with $\langle \phi_1 \rangle = (0, 0, V)^T$, $\phi_2(1, 3^*, -1/3)$ with $\langle \phi_2 \rangle = (0, v\sqrt{2}, 0)^T$, $\phi_3(1, 3^*, 2/3)$ with $\langle \phi_3 \rangle = (v'/\sqrt{2}, 0, 0)^T$ and $V > v \sim v'$.^{11,12}

Our main aim concerns the leptonic phenomenology and therefore only the leptonic sector will be addressed. The Lagrangian for the neutral currents in this sector is

$$\begin{aligned} \mathcal{L}_{\text{NC}} = & - \sum_{\ell} \left[g S_W A_{\mu} \left\{ \bar{\ell}^0 \gamma_{\mu} \epsilon_{\ell_L}^A P_L \ell^0 + \bar{\ell}^0 \gamma_{\mu} \epsilon_{\ell_R}^A P_R \ell^0 \right\} \right. \\ & + \frac{g Z^{\mu}}{2 C_W} \left\{ \bar{\ell}^0 \gamma_{\mu} \epsilon_{\ell_L}^Z P_L \ell^0 + \bar{\ell}^0 \gamma_{\mu} \epsilon_{\ell_R}^Z P_R \ell^0 \right\} \\ & \left. + \frac{g' Z'^{\mu}}{2\sqrt{3} S_W C_W} \left\{ \bar{\psi}^0 \gamma_{\mu} \epsilon_{\ell_L}^{Z'} P_L \ell^0 + \bar{\ell}^0 \gamma_{\mu} \epsilon_{\ell_R}^{Z'} P_R \ell^0 \right\} \right], \end{aligned} \tag{8}$$

where ℓ^0 in this notation stands for the charged leptons vector $\ell^{0T} = (e_1^{0-}, e_2^{0-}, e_3^{0-}, E_1^{0-}, E_2^{0-})$. The zero superscript denotes that the fields are in the interaction basis, and the couplings to the neutral bosons are

$$\begin{aligned} \epsilon_{\ell_L}^A &= I_{5 \times 5}, \\ \epsilon_{\ell_{(R)}}^A &= I_{5 \times 5}, \\ \epsilon_{\ell_L}^Z &= \operatorname{diag} (C_{2W}, C_{2W}, C_{2W}, -2S_W^2, C_{2W}), \\ \epsilon_{\ell_R}^Z &= \operatorname{diag} (-2S_W^2, -2S_W^2, -2S_W^2, -2S_W^2, C_{2W}), \\ \epsilon_{\ell_L}^{Z'} &= \operatorname{diag} (1, -C_{2W}, -C_{2W}, -C_{2W}, -C_{2W}), \\ \epsilon_{\ell_R}^{Z'} &= \operatorname{diag} (2S_W^2, 2S_W^2, -C_{2W}, 2S_W^2, 1), \end{aligned} \tag{9}$$

where $C_{2W} = \cos(2\theta_W)$. Note that the couplings of the standard charged leptons to the photon A_{μ} are universal, as well as the couplings to the Z boson. A feature of this model is that the couplings of the standard left-handed leptons as well as the right-handed leptons to the Z' boson are not universal, due to the fact that one

of the lepton triplets is in a different representation to the other two. Since these couplings to the Z' boson are not universal, at least for the standard leptons, when they are rotated to mass eigenstates, the obtained mixing matrix will allow LFV at tree level.

A similar procedure in the neutral leptonic sector can be done, $N^{0T} = (\nu_1^0, \nu_2^0, \nu_3^0, N_1^0, N_2^0, N_3^0, N_4^0, N_5^0)$ generating the couplings

$$\begin{aligned} \epsilon_{N_L}^A &= 0, \\ \epsilon_{N_L}^Z &= \text{diag}(1, 1, 1, 0, 0, 1, 0, -1), \\ \epsilon_{N_L}^{Z'} &= \text{diag}\left(1, -C_{2W}, -C_{2W}, 2C_W^2, 2C_W^2, -C_{2W}, 2C_W^2, -1\right), \end{aligned} \tag{10}$$

where the couplings of the standard neutrinos to the photon A and Z boson are universal, but the couplings of the corresponding leptons to the Z' are not.

It is possible to rewrite the neutral current Lagrangian in order to use the formalism presented in Ref. 54 and generate an effective Lagrangian like

$$\mathcal{L}_{\text{NC}}^{\text{eff}} = -eJ_{\text{em}}^\mu A_\mu - g_1 J^{(1)\mu} Z_{1\mu} - g_2 J^{(2)\mu} Z_{2\mu}, \tag{11}$$

where the currents associated to the gauge Z and Z' bosons are

$$J_\mu^{(1)} = \sum_{ij} \bar{\ell}_i^0 \gamma_\mu \left(\epsilon_{\ell_L}^Z P_L + \epsilon_{\ell_R}^Z P_R \right) \ell_j^0, \tag{12}$$

$$J_\mu^{(2)} = \sum_{ij} \bar{\ell}_i^0 \gamma_\mu \left(\epsilon_{\ell_L}^{Z'} P_L + \epsilon_{\ell_R}^{Z'} P_R \right) \ell_j^0, \tag{13}$$

with $g_1 = g/C_W$. The ℓ_i^0 leptons and the gauge bosons Z_1 and Z_2 are interaction eigenstates and the matrices $\epsilon_{\ell_{L,R}}^Z$ and $\epsilon_{\ell_{L,R}}^{Z'}$ in the charged sector were defined in Eq. (9). When the fields of the theory are rotated to mass or physical eigenstates, the effective Lagrangian for the charged leptons can be finally written as:

$$\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} \sum_{ijkl} \sum_{XY} C_{XY}^{ijkl} (\bar{\ell}_i \gamma^\mu P_X \ell_j) (\bar{\ell}_k \gamma_\mu P_Y \ell_l), \tag{14}$$

where X and Y run over the chiralities L, R and indices i, j, k, l over the leptonic families. The coefficients C_{XY}^{ijkl} for the standard leptons, assuming a mixing angle θ between Z and Z' bosons, are given by,⁵⁴

$$C_{XY}^{ijkl} = z\rho \left(\frac{g_2}{g_1} \right)^2 B_{ij}^X B_{kl}^Y, \tag{15}$$

where

$$\rho = \frac{m_W^2}{m_{Z'}^2 C_W^2}, \quad z = \left(\sin^2 \theta + \frac{m_Z^2}{m_{Z'}^2} \cos^2 \theta \right), \quad \left(\frac{g_2}{g_1} \right)^2 = \frac{1}{3(1 - 4S_W^2)}. \tag{16}$$

The B^X corresponds to the matrices obtained when the unitary matrices $V_{L,R}^\ell$ are introduced to obtain the mass eigenstates and to diagonalize the Yukawa coupling matrices:

$$B^X = V_X^{\ell\dagger} \epsilon_{\ell}^{Z'} V_X^\ell. \tag{17}$$

For the matrix V , we will use a well-accepted ansatz⁵⁵ where

$$V_L^\ell = P\tilde{V}K, \tag{18}$$

with $P = \text{diag}(e^{i\phi_1}, 1, e^{i\phi_3})$, $K = \text{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3})$, and the unitary matrix \tilde{V} can be parametrized using three standard mixing angles θ_{12} , θ_{23} and θ_{13} and a phase φ ,

$$\tilde{V} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\varphi} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\varphi} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\varphi} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\varphi} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\varphi} & c_{23}c_{13} \end{pmatrix}. \tag{19}$$

Note that, if we are considering only the standard charged leptons, the coupling matrices in Eq. (9) can be written as

$$\begin{aligned} \epsilon_{\ell_L}^{Z'} &= -(1 - 2S_W^2)\mathbf{I}_{3\times 3} + 2C_W^2 \text{diag}(1, 0, 0), \\ \epsilon_{\ell_R}^{Z'} &= 2S_W^2\mathbf{I}_{3\times 3} - \text{diag}(0, 0, 1). \end{aligned} \tag{20}$$

The terms which are proportional to identity are not contributing to the LFV processes at tree level, while the second term in the above equations does. These equations (20) correspond to the case where the first family is in the adjoint representation. However, if the second family was the chosen one to be in a different representation, then the only change is in the second term which is proportional to $\text{diag}(0, 1, 0)$. Finally, if instead of that the third family was chosen, then again the only change is the position of the entry different from zero in the second term. We should emphasize that the source of LFV in neutral currents mediated by the Z' boson, comes from the nondiagonal elements in the 3×3 matrices $B_{L,R}^\ell$.

3. LFV Processes

Our next task is to get bounds on the parameters involved in the LFV couplings and it is done considering different LFV processes. Recently, the BELLE⁵⁶⁻⁶⁰ and BABAR⁶¹⁻⁶³ collaborations have reported measurements of various LFV channels and they have put new bounds on these branching fractions, see Table 1. Other channels to consider are $\text{Br}(\mu^- \rightarrow e^- \gamma) < 2, 4 \times 10^{-12}$ (Refs. 56-60) and $\text{Br}(\mu^- \rightarrow e^- e^- e^+) < 1, 0 \times 10^{-12}$ (Refs. 56-60).

In the framework of the model 331 that we presented in Sec. 2, we calculated the decay widths for the different processes that we are going to consider. For the $l_j \rightarrow l_i \gamma$ processes, the decay widths are

$$\Gamma(l_j \rightarrow l_i \gamma) = \frac{\alpha G_F^2 M_j^3}{8\pi^4} \left(\frac{g_2}{g_1}\right)^4 \rho^2 \left[(B^R M_l B^L)_{ij}^2 + (B^L M_l B^R)_{ij}^2 \right], \tag{21}$$

with $i, j = e, \mu, \tau$ and M_l a diagonal mass matrix where the electron mass has been neglected. From Table 1, we should also evaluate the decay widths into three

Table 1. Experimental data and their bounds from BELLE⁵⁶⁻⁶⁰ and BABAR⁶¹⁻⁶³

Processes	Br($\times 10^{-8}$) BELLE	Br($\times 10^{-8}$) BABAR
$\tau^- \rightarrow e^- \gamma$	12	3.3
$\tau^- \rightarrow \mu^- \gamma$	4.5	4.4
$\tau^- \rightarrow e^- e^+ e^-$	2.7	2.9
$\tau^- \rightarrow \mu^- \mu^+ \mu^-$	2.1	3.3
$\tau^- \rightarrow e^- \mu^+ \mu^-$	2.7	3.2
$\tau^- \rightarrow \mu^- e^+ e^-$	1.8	2.2
$\tau^- \rightarrow e^+ \mu^- \mu^-$	1.7	2.6
$\tau^- \rightarrow \mu^+ e^- e^-$	1.5	1.8

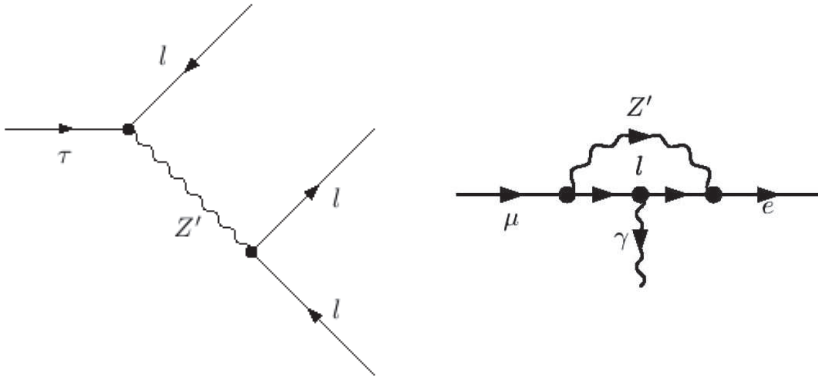


Fig. 1. Feynman diagram for the tau (muon) lepton decay into three body through the new Z' current and the diagram for $\mu^- \rightarrow e^- \gamma$.

charged leptons, see Fig. 1,

$$\begin{aligned}
 \Gamma(l_j \rightarrow l_i^- l_i^- l_i^+) &= \frac{G_F^2 M_{l_j}^5}{48\pi^3} \left(\frac{g_2}{g_1}\right)^4 \rho^2 \\
 &\quad \times \left[2|B_{ij}^L B_{ii}^L|^2 + 2|B_{ij}^R B_{ii}^R|^2 + |B_{ij}^L B_{ii}^R|^2 + |B_{ij}^R B_{ii}^L|^2 \right], \\
 \Gamma(l_j \rightarrow l_i^- l_k^- l_l^+) &= \frac{G_F^2 M_{l_j}^5}{48\pi^3} \left(\frac{g_2}{g_1}\right)^4 \rho^2 \\
 &\quad \times \left[|B_{ij}^L B_{kl}^L + B_{kj}^L B_{il}^L|^2 + |B_{ij}^R B_{kl}^R + B_{kj}^R B_{il}^R|^2 \right. \\
 &\quad \left. + |B_{ij}^L B_{kl}^R|^2 + |B_{kj}^L B_{il}^R|^2 + |B_{ij}^R B_{kl}^L|^2 + |B_{kj}^R B_{il}^L|^2 \right],
 \end{aligned} \tag{22}$$

where the elements $B_{ij}^{L,R}$ are defined in Eq. (17) and ρ in Eq. (16).

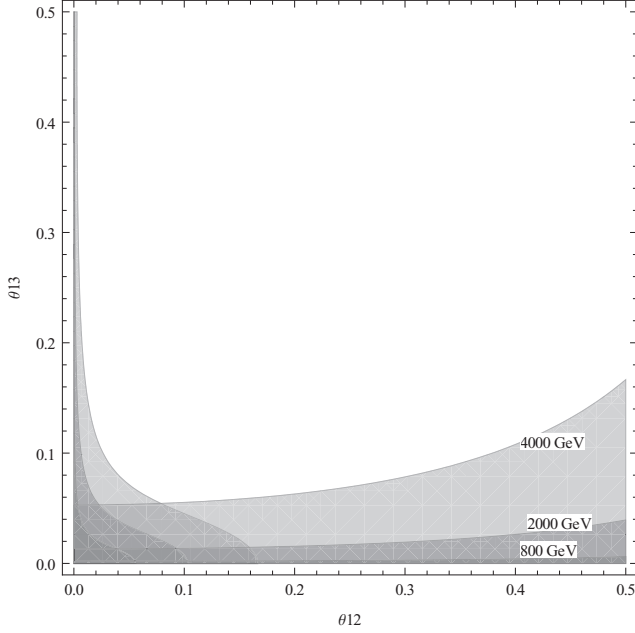


Fig. 2. The allowed region from processes $\tau \rightarrow lll$ and $\mu \rightarrow e\gamma$ in the $\theta_{12} - \theta_{13}$ plane using different Z' boson masses (800, 2000 and 4000 GeV).

In order to do the numerical analysis, we trace back the final parameters which are going to be present in the decay widths, namely the mixing angles θ_{12} , θ_{23} , θ_{13} and the Z' gauge boson mass. There are also phases coming from the V_L^l matrix. We have found their effect to be negligible and therefore we have assumed them equal to zero. We are going to consider two cases depending on which leptonic family is in a different representation of $SU(3)_L$: the first or the third leptonic family. We should mention that the option of the second leptonic family in a different representation is completely analogous to the case of the first family, so we do not present that case.

For the case of the first leptonic family in a different representation, the rotation matrix in the charged leptonic sector depends on θ_{12} , θ_{13} and the Z' boson mass, assuming that the phases involved are zero. Now, we use the experimental bounds on different LFV processes shown in Table 1 in order to obtain constraints on the mixing parameters and the Z' boson mass. In general, the observables considered here are proportional to ρ^2 , Eq. (16), which is depending on the Z' gauge boson mass resulting in a dominant factor $\sim m_{Z'}^{-4}$ in Eqs. (21) and (22). In Fig. 2, bounds coming for the six decay widths of τ into three charged leptons are shown in the $\theta_{12} - \theta_{13}$ plane, the allowed regions plotted are covering the right side of the plane. We have used Z' boson masses of 800, 2000 and 4000 GeV. On the other hand, from the process $\tau \rightarrow e\gamma$, it is observed that for $\theta_{12} < 0.1$ the mixing angle θ_{13} could be up to (0.08, 0.14, 0.2) for the Z' boson masses $m_{Z'} = (800, 2000, 4000)$ GeV, it is corresponding to the zones allowed in the left side of the plot. Finally, the processes

$\mu \rightarrow eee$ and $\tau \rightarrow \mu\gamma$ are not generating stronger bounds on the parameters than the ones mentioned previously.

Now, the case of the third family transforming differently to the other two families using masses for the Z' gauge boson of 800, 2000 and 4000 GeV. In Fig. 3, it is shown the results from the process $\mu \rightarrow e\gamma$ with the three mixing angles, we are going to take one of them zero each time and show the plane of the other two angles. In the case of $\theta_{23} = 0$ then the plane $\theta_{12} - \theta_{13}$ is plotted (left figure), there the mixing angle θ_{12} does not get any bound in this range from this observable and the angle θ_{13} should be of the order of 10^{-2} – 10^{-3} for the Z' masses considered. Taking $\theta_{13} = 0$, then the plane $\theta_{12} - \theta_{23}$ is shown (center figure), the mixing angles $\theta_{12} - \theta_{23}$ should be of the order of 10^{-1} . And finally considering $\theta_{12} = 0$ then the plane $\theta_{23} - \theta_{13}$ is shown (right figure), again the other two mixing angles are of order of 10^{-1} . In Fig. 4, the $\mu \rightarrow eee$ decay is considered. For this decay, taking the cases $\theta_{13} = 0$ and $\theta_{23} = 0$, any improved bound is obtained, assuming small mixing angles. But for the case $\theta_{12} = 0$, the plane $\theta_{23} - \theta_{13}$ shown, the mixing angles are of the order of $\theta_{23} \sim 10^{-2}$ and $\theta_{13} \sim 10^{-3}$. Figure 5 is considering the decay $\tau \rightarrow lll$ which are six different decays, see Table 1. Following the same analysis, when $\theta_{23} = 0$ there is no strong dependence on θ_{12} while the mixing angle θ_{13} is of the order of $\sim 10^{-1}$ (left figure). Similarly with $\theta_{13} = 0$, there is no sensitivity to θ_{12} but $\theta_{23} \sim 10^{-1}$ (center figure). And when $\theta_{12} = 0$ is considered, then the other two mixing angles remain in the same order of magnitude and they only change with the Z' gauge boson mass (right figure). Finally, in Fig. 6, bounds using the $\tau \rightarrow \mu\gamma$ are obtained. We have noticed that taking $\theta_{12} = 0$ and $\theta_{23} = 0$, there are not any improved bounds on the parameters and for the case $\theta_{13} = 0$, the mixing angles θ_{12} and θ_{23} are in the same order of magnitude. We should mention that using the process $\tau \rightarrow e\gamma$, that has not obtained any bound on the mixing angles lower than the previously obtained, so it is less restrictive than the other processes considered here. The whole set of bounds obtained in Figs. 2–5 help to obtain the order of magnitude of the mixing angles involved in order to satisfy the experimental bounds on the LFV processes considered in this paper. There is one order of magnitude of difference between them and the hierarchy $\theta_{12} \sim 10^{-1} > \theta_{23} \sim 10^{-2} > \theta_{13} \sim 10^{-3}$.

Now, we can use the information obtained about the mixing angles and see what is happening in the neutrino sector of the model. First of all, we write down the charged current Lagrangian as:

$$\mathcal{L}_{cc} = -\frac{g}{\sqrt{2}}(\bar{\nu}_L^0 \gamma^\mu \ell_L^0)W_\mu^+ + \text{h.c.}, \quad (23)$$

where quantum fields are in the weak interaction basis. In order to go to the mass eigenstates, it is necessary to introduce the rotation matrices V_L^ν and V_L^ℓ which are also diagonalizing the Yukawa matrices. These matrices are defining the mixing matrix in the leptonic sector known as Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix and it is $V_{\text{PMNS}} = V_L^\nu (V_L^\ell)^\dagger$. The matrices V_L^ν and V_L^ℓ are parametrized as in Eq. (19). Now taking into account that $\theta_{12} \sim 10^{-1} > \theta_{23} \sim$

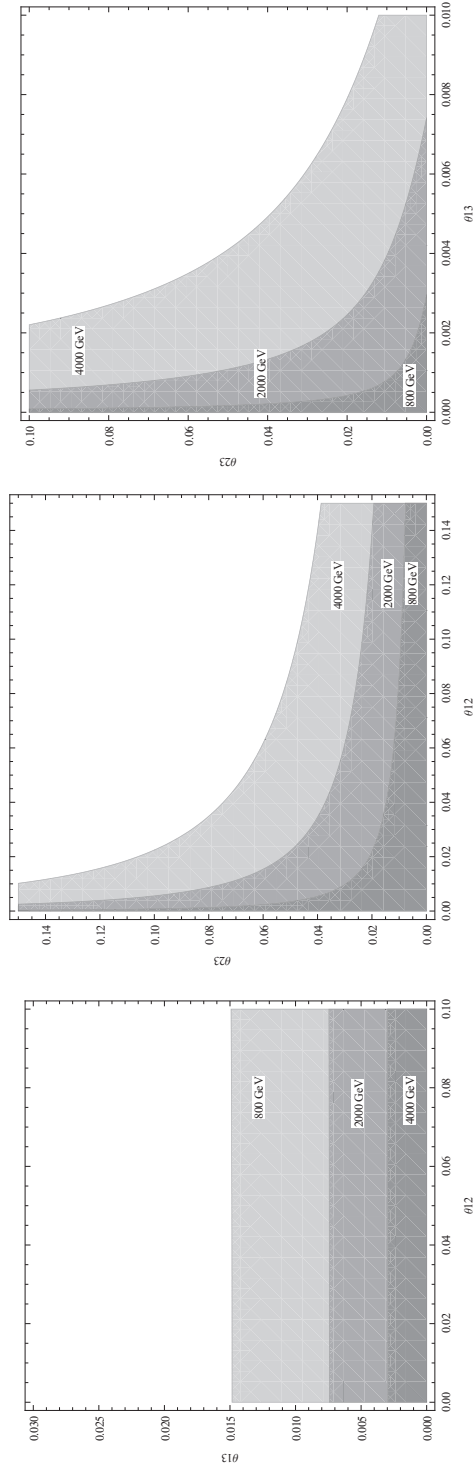


Fig. 3. Bounds coming from the $\mu \rightarrow e\gamma$ process in the different planes such that the third mixing angle is set to zero for $m_{Z'} = 800, 2000$ and 4000 GeV.

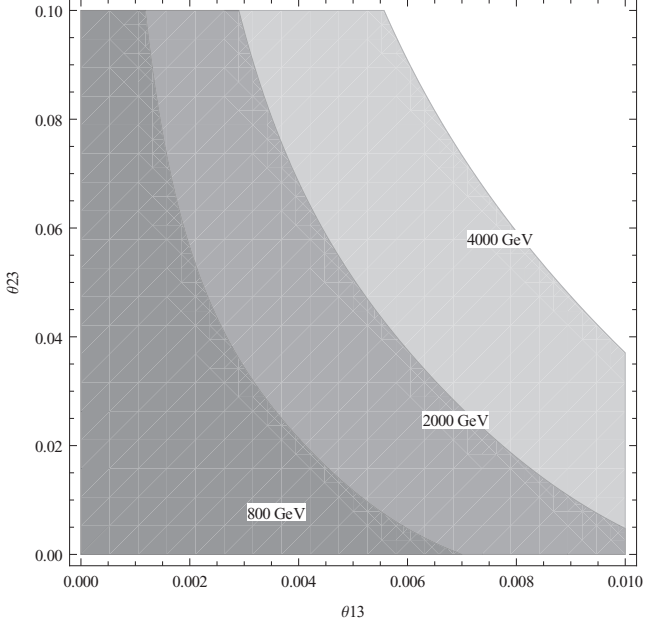


Fig. 4. Bounds from $\mu \rightarrow eee$ in the $\theta_{13} - \theta_{23}$ plane using $m_{Z'} = 800, 2000$ and 4000 GeV.

$10^{-2} > \theta_{13} \sim 10^{-3}$, it is possible to obtain the following limits on the mixing leptonic matrix

$$|V_L^\ell| = \begin{pmatrix} 1 \rightarrow 0.995004 & 0 \rightarrow 0.0998334 & 0 \rightarrow 0.001 \\ 0 \rightarrow 0.0998384 & 1 \rightarrow 0.994953 & 0 \rightarrow 0.00999983 \\ 0 \rightarrow 3.36328 \times 10^{-6} & 0 \rightarrow 0.0100497 & 1 \rightarrow 0.99995 \end{pmatrix}. \quad (24)$$

On the other hand, considering massive neutrinos and the oscillation data, the mixing angles in the neutrino sector are around $\theta_{12} \sim 30^\circ$, $\theta_{23} \sim 45^\circ$ and $\theta_{13} \sim 8^\circ$,^{20,64-67} where the final mixing matrix is (at 3σ C.L.)

$$|V_L^\nu| = \begin{pmatrix} 0.795 \rightarrow 0.846 & 0.513 \rightarrow 0.585 & 0.126 \rightarrow 0.178 \\ 0.205 \rightarrow 0.543 & 0.416 \rightarrow 0.730 & 0.579 \rightarrow 0.808 \\ 0.215 \rightarrow 0.548 & 0.409 \rightarrow 0.725 & 0.567 \rightarrow 0.800 \end{pmatrix}. \quad (25)$$

Finally, we can combine the matrices to obtain the PMNS matrix and it is

$$|V_{\text{PMNS}}| = \begin{pmatrix} 0.795 \rightarrow 0.900 & 0.513 \rightarrow 0.668 & 0.126 \rightarrow 0.183 \\ 0.205 \rightarrow 0.613 & 0.416 \rightarrow 0.788 & 0.579 \rightarrow 0.815 \\ 0.215 \rightarrow 0.618 & 0.409 \rightarrow 0.784 & 0.567 \rightarrow 0.807 \end{pmatrix}, \quad (26)$$

which is in agreement with the accepted values for this matrix in the literature.^{20,64-67}

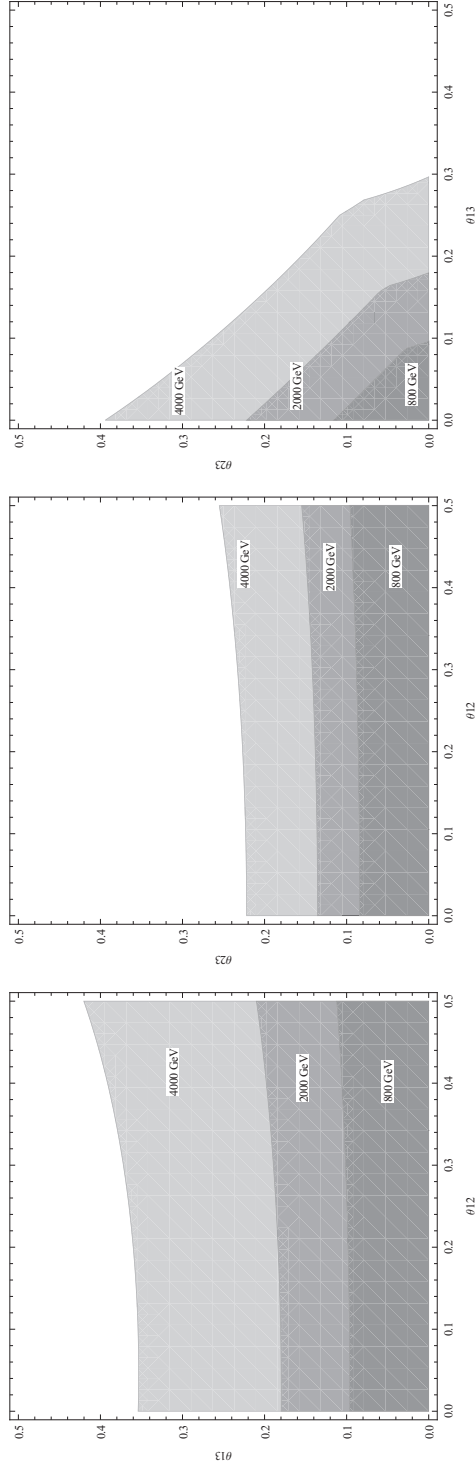


Fig. 5. Bounds obtained using the $\tau \rightarrow lll$ processes of Table 1, with the three different scenarios described in Sec. 3 for θ_{12} , θ_{13} and θ_{23} with $m_{Z'} = 800, 2000$ and 4000 GeV.

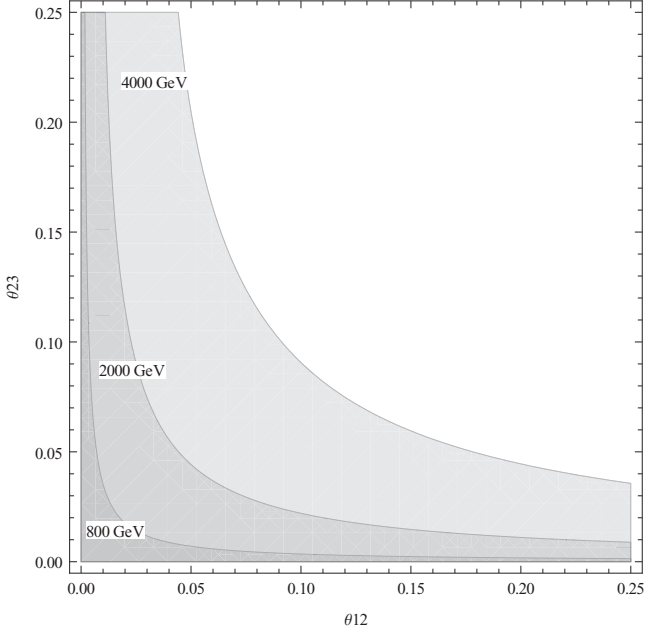


Fig. 6. Bounds from $\tau \rightarrow \mu\gamma$ in the $\theta_{12} - \theta_{23}$ plane with $m_{Z'} = 800, 2000$ and 4000 GeV.

4. Conclusions

In this paper, we have addressed the LFV processes in a model based on the 331 symmetry where the leptonic sector is described by five left-handed leptonic triplets in different representations of the $SU(3)_L$ gauge group. Here, the couplings of the new neutral Z' boson with the usual leptons are not universal. This feature is due to one of the lepton triplets being in a different representation than the other two, which leads to LFV at tree level once they are rotated to mass eigenstates. We have considered some LFV processes which have been measured by the BELLE and the BABAR collaborations: $\tau \rightarrow 3l$, $\tau \rightarrow l\gamma$, $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$ (see Table 1). The analysis was done considering two cases depending on which leptonic family is in the different representation of $SU(3)_L$ in the 331 model described in Sec. 2. For the first case (where the first leptonic family is in the different representation), we obtained allowed regions on the $\theta_{12} - \theta_{23}$ plane of the order of $\sim 10^{-1}$. For the second case (the third leptonic family in a different representation), the bound on the process $\mu \rightarrow e\gamma$ constrain the space of parameters to regions around $\theta_{12} \sim 10^{-2}$, $\theta_{23} \sim 10^{-2}$ and $\theta_{13} \sim 10^{-3}$. We also explored the bounds coming from other LFV processes, which are consistent with these regions and the results are shown in Figs. 2–5. It is worth to point out that the mixing angles obtained for the leptons in the framework of the 331 model considered here is generating a matrix which is almost an identity matrix which is according with the no experimental evidence of the FCNC at low energies. Therefore, even considering the mixing in the neutrino

sector, the results are in agreement with the experimental values reported for the PMNS matrix.

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