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Electric charge quantization in $\mathbf{SU(3)_c \otimes SU(4)_L \otimes U(1)_X}$ models

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We obtain electric charge quantization in the context of models based on the gauge symmetry group $\mathrm{SU}(3)_c \otimes \mathrm{U}(4)_L \otimes U(1)_X$. The gauge models study include three families to cancel out anomalies and a set of scalar fields to break the symmetry spontaneously. To show the electric charge quantization, we use classical symmetry conditions and quantum chiral anomaly conditions.

Keywords: Beyond the Standard Model; charge quantization.

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One of the most intriguing questions in the last decade is concerning the electric charge quantization. It remains as an open question, although there are some proposals. The first one given by Dirac, includes magnetic monopoles in a quantum mechanical theory implying that electric charge is quantized.[1] Another one came from grand unification theory using the group structure itself. It is based on gauge models, that contain explicitly the U(1) Abelian group in their structure and contributes to the $\mathrm{U}(1)_{em}$ after the spontaneous symmetry breaking.[2,3] Finally, using this idea, some studies have been done in the framework of the $\mathrm{SU}(3)_c \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X$ (331) models.[4,5] They use classical and quantum constraints to obtain the relationships between the U(1) charges and lead to the electric charge quantization. It is relevant that in the standard model with one family the electric charge quantization can be obtained, but when the three families are considered with massless neutrinos, then a dequantization arises up. It is possible to

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restore the electric charge quantization, if Majorana neutrinos are included. This is related with the global hidden symmetry $\mathrm{U}(1)_{B-L}$. On the other hand, in the framework of the 331 models the electric charge quantization is obtained when three families are involved all together and it does not depend on the neutrino mass. Moreover, if neutrinos are massive, the charge electric quantization does not depend on the neutrino type, i.e. Dirac or Majorana type.[4,5]

The model 331 which enlarge the gauge group of the standard model share with the $\mathrm{SU}(3)_c \otimes \mathrm{SU}(4)_L \otimes \mathrm{U}(1)_X$ (341) version, the interesting feature of addressing the problem of the number of fermion families.[6–15] It is concern to the anomaly cancellation among the families which is obtained when the number of left-handed triplets is equal to the antitriplets in the 331 model[16–25] and equal number of 4-plets and 4*-plets in the 341 model.[6,8–13] Taking into account the color degree of freedom. On the other hand, if we add a right-handed neutrino, one option is to have ν, e^-, ν^c and e^c in the same multiplet of $\mathrm{SU}(4)_L$. The gauge group SU(4) is the highest symmetry group to be considered in the electroweak sector as a consequence of using the lightest leptons as the particles which determine the gauge symmetry, each generator are treated separately. Models based on the gauge symmetry $\mathrm{SU}(3)_c \otimes \mathrm{SU}(4)_L \otimes \mathrm{U}(1)_X$ have been studied before[9] and also this symmetry appears in some Little Higgs models.[26–29] The issue of the quantization in the 341 models has been studied only in a model with bilepton–leptoquarks in the spectrum.[30] But, for instance, has not been generalized to models without exotic charges in the spectrum and even more its relationship with the character (Majorana or Dirac) of the neutrino masses is not so clear. Therefore, it is interesting to look for its own how quantization is working in the framework of different models based on the gauge symmetry 341. In this paper, we study models based on $\mathrm{SU}(3)_c \otimes \mathrm{SU}(4)_L \otimes \mathrm{U}(1)_X$ gauge group in order to show how the electric charge quantization is satisfied using classical and quantum conditions.

We are going to consider models based on the gauge group $\mathrm{SU}(3)_c \otimes \mathrm{SU}(4)_L \otimes \mathrm{U}(1)_X$. The electric charge operator is defined as a linear combination of the diagonal generators of the group

$$Q = T_3 + \beta T_8 + \gamma T_{15} + X_f \,, \tag{1}$$

where $T_i = \lambda_i/2$ with λ_i the Gell-Mann matrices for SU(4) and X_f the quantum number associated to $\mathrm{U}(1)_X$. And the parameters β and γ define the spectrum of the models, as we are going to show later on. Usually in these models, in order to break spontaneously the gauge symmetry and give masses to the quark sector is necessary a set of four scalars

$$\chi \sim (1, 4, X_\chi)\,, \qquad \xi \sim (1, 4, X_\xi)\,, \qquad \eta \sim (1, 4, X_\eta) \qquad \text{and} \qquad \rho \sim (1, 4, X_\rho)\,, \tag{2}$$

with the following vacuum expectation values (VEVs)

$$\begin{aligned} \langle\chi\rangle_0 &= \begin{pmatrix}0 & 0 & 0 & W/\sqrt{2}\end{pmatrix}^T, \qquad \langle\xi\rangle_0 = \begin{pmatrix}0 & 0 & V/\sqrt{2} & 0\end{pmatrix}^T, \\ \langle\eta\rangle_0 &= \begin{pmatrix}0 & w/\sqrt{2} & 0 & 0\end{pmatrix}^T, \qquad \langle\rho\rangle_0 = \begin{pmatrix}v/\sqrt{2} & 0 & 0 & 0\end{pmatrix}^T. \end{aligned} \tag{3}$$

To verify that the electric charge operator annihilates the VEVs to have electric charge conservation, we obtain

$$\begin{aligned}
X_\chi &= \frac{3\gamma}{2\sqrt{6}}\,,\\
X_\xi &= \frac{\beta}{\sqrt{3}} - \frac{\gamma}{2\sqrt{6}}\,,\\
X_\eta &= \frac{1}{2} - \frac{\beta}{2\sqrt{3}} - \frac{\gamma}{2\sqrt{6}}\,,\\
X_\rho &= -\frac{1}{2} - \frac{\beta}{2\sqrt{3}} - \frac{\gamma}{2\sqrt{6}}\,,
\end{aligned} \tag{4}$$

which also satisfies the relationship $X_\chi + X_\xi + X_\eta + X_\rho = 0$.

On the other hand, the mass Lagrangian for the neutral gauge bosons $V^T = \begin{pmatrix} W^3_\mu & W^8_\mu & W^{15}_\mu & B_\mu \end{pmatrix}$ is

$$\mathcal{L}_{\text{mass}} = \frac{1}{2} V^T M^2 V\,. \tag{5}$$

As it is usual, there is a zero non-degenerate eigenvalue of the matrix M^2 which is identified with the photon field A_μ. In particular, the associated eigenvector is

$$A_\mu = \frac{g}{\sqrt{g^2 + (1+\beta^2+\gamma^2)g'^2}} \left(\frac{g'}{g} W^3_\mu + \beta \frac{g}{g} W^8_\mu + \gamma \frac{g'}{g} W^{15}_\mu + B_\mu \right) \tag{6}$$

but we also have

$$\begin{pmatrix} A_\mu & Z^1_\mu & Z^2_\mu & Z^3_\mu \end{pmatrix}^T = \begin{pmatrix} W^3_\mu & W^8_\mu & W^{15}_\mu & B_\mu \end{pmatrix}^T U^T\,. \tag{7}$$

where U is 4×4 rotation matrix which can be written as

$$U = \begin{pmatrix} \frac{g'}{r} & U_{12} & U_{13} & U_{14} \\ \frac{\beta g'}{r} & U_{22} & U_{23} & U_{24} \\ \frac{\gamma g'}{r} & U_{32} & U_{33} & U_{34} \\ \frac{g}{r} & U_{42} & U_{43} & U_{44} \end{pmatrix}, \tag{8}$$

with $r = \sqrt{g^2 + (1+\beta^2+\gamma^2)g'^2}$. The elements U_{ij} , $(i, j = 2, 3, 4)$ are not relevant in this study.

Now, Eq. (6) for the photon field implies that for an up-quark type in the fundamental representation, the Lagrangian can be written as:

$$\begin{aligned}\mathcal{L}_{\bar{u}u\gamma} &= \bar{u}_L i\gamma^\mu \left[\frac{ig}{2}U_{11} + \frac{ig}{2\sqrt{3}}U_{21} + \frac{ig}{2\sqrt{6}}U_{31} + ig' X_{4q}U_{41}\right] A_\mu u_L \\ &\quad + \bar{u}_R i\gamma^\mu \left[ig' X_u^R \frac{g}{r}\right] A_\mu u_R \\ &= -\frac{gg'}{r}\left[\frac{1}{2} + \frac{\beta}{2\sqrt{3}} + \frac{\gamma}{2\sqrt{6}} + X_{4q}\right]\bar{u}_L\gamma^\mu u_L A_\mu - \frac{gg'}{r}X_u^R \bar{u}_R\gamma^\mu u_R A_\mu \,. \end{aligned} \quad (9)$$

But taking into account $X_\rho = -\frac{1}{2} - \frac{\beta}{2\sqrt{3}} - \frac{\gamma}{2\sqrt{6}}$, the Lagrangian takes the form

$$\mathcal{L}_{\bar{u}u\gamma} = -\frac{gg'}{r}[X_{4q} - X_\rho]\bar{u}_L\gamma^\mu u_L A_\mu - \frac{gg'}{r}[X_u^R]\bar{u}_R\gamma^\mu u_R A_\mu \quad (10)$$

and therefore asking for invariance under parity transformations, we arrive to $X_u^R = X_{4q} - X_\rho$. Using the same arguments, we obtain for the heavy sector

$$\begin{aligned} X_d^R &= X_{4q} - X_\eta \,, \\ X_J^R &= X_{4q} - X_\xi \,, \\ X_{\tilde{J}}^R &= X_{4q} - X_\chi \,. \end{aligned} \quad (11)$$

On the other hand, quarks can also be in the adjoint representation and for that case, we obtain

$$\begin{aligned} X_{u'}^R &= X_{4q*} + X_\eta \,, \\ X_{d'}^R &= X_{4q*} + X_\rho \,, \\ X_{J'}^R &= X_{4q*} + X_\xi \,, \\ X_{\tilde{J}'}^R &= X_{4q*} + X_\chi \,. \end{aligned} \quad (12)$$

For the leptonic sector, the electromagnetic Lagrangian will be

$$\begin{aligned}\mathcal{L}_{\bar{\nu}\nu\gamma} &= \bar{\nu}_L i\gamma^\mu \left[\frac{ig}{2}U_{11} + \frac{ig}{2\sqrt{3}}U_{21} + \frac{ig}{2\sqrt{6}}U_{31} + ig' X_{4\ell}U_{41}\right] A_\mu \nu_L \\ &= -\frac{gg'}{r}\left[\frac{1}{2} + \frac{\beta}{2\sqrt{3}} + \frac{\gamma}{2\sqrt{6}} + X_{4\ell}\right]\bar{\nu}_L\gamma^\mu \nu_L A_\mu \,, \end{aligned} \quad (13)$$

implying the relationship $X_{4\ell} = -\frac{1}{2} - \frac{\beta}{2\sqrt{3}} - \frac{\gamma}{2\sqrt{6}} = X_\rho$ under parity invariance. Similarly for the other leptons including the heavy ones, we obtain

$$\begin{aligned} X_\ell^R &= X_{4\ell} - X_\eta \,, \\ X_{F_\alpha}^R &= X_{4\ell} - X_\xi \,, \\ X_{\tilde{F}_\alpha}^R &= X_{4\ell} - X_\chi \,. \end{aligned} \quad (14)$$

Up to now, we have dealt with classical symmetry conditions but we have to also consider the conditions coming from the vanishing of the chiral anomaly coefficients. The relevant and nontrivial conditions in our particular case should satisfy

$$\begin{aligned}
&\sum X_\ell^L + 3\sum X_q^L = 0\,,\\
&3\sum X_q^L - \sum_{\text{sing}} X_q^R = 0\,,\\
&4\sum X_\ell^L + 12\sum X_q^L - 3\sum_{\text{sing}} X_q^R - \sum_{\text{sing}} X_\ell^R = 0\,,\\
&4\sum (X_\ell^L)^3 + 12\sum (X_q^L)^3 - 3\sum_{\text{sing}} (X_q^R)^3 - \sum_{\text{sing}} (X_\ell^R)^3 = 0\,.
\end{aligned} \tag{15}$$

Considering explicitly the first equation (15), we obtain

$$3X_{4\ell}^L + 3(X_{4q}^L - 2X_{4q*}^L) = 0 \tag{16}$$

and using $X_{4\ell}^L = X_\rho$ then $X_{4q}^L = 2X_{4q*} - X_\rho$. Additionally taking into account Eqs. (11) and (12), we get $X_{4q}^L + X_{4q*}^L = X_\rho - X_\eta$ and therefore

$$\begin{aligned}
X_{4q*}^L &= \frac{2X_\rho - X_\eta}{3}\,,\\
X_{4q}^L &= \frac{X_\rho + 2X_\eta}{3}\,.
\end{aligned} \tag{17}$$

With a similar procedure, we obtain the following relations for the quark sector

$$\begin{aligned}
X_u^R &= \frac{2(X_\eta - X_\rho)}{3}\,,\\
X_d^R &= \frac{(X_\rho - X_\eta)}{3}\,,\\
X_J^R &= \frac{X_\rho + 2X_\eta - 3X_\xi}{3}\,,\\
X_{\tilde{J}}^R &= \frac{X_\rho + 2X_\eta - 3X_\chi}{3}\,.
\end{aligned} \tag{18}$$

These results are relevant for the electric charges because of the X quantum numbers dependence on the scalar fields. Implying that the fermion electric charges are been quantized as a function of the scalar fields quantum numbers.

In the literature, many models based on the gauge symmetry 341 have been studied and main differences are found on how the fermions are assigned in the possible group representations.[6,8,9] Different values of the parameters β and γ in the charge operator (1) are fixed depending on how the fermions are in the multiplets of the group. The criterion to classify the possible models based on 341 symmetry is given by the values of the parameters β and γ, generating models with or without exotic electric charges.[8] Now, we consider the different associated fermion representations for the 341 gauge symmetry models found in the literature. In general, fermions are given by

$$\begin{aligned} q_i^T &= \begin{pmatrix} d_i & u_i & J_i & \tilde{J}_i \end{pmatrix}_L \sim (1, \mathbf{4}*, X_{4q}) \,, \\ q_3^T &= \begin{pmatrix} u_3 & d_3 & J_3 & \tilde{J}_3 \end{pmatrix}_L \sim (1, \mathbf{4}, X_{4q*}) \,, \\ l_\alpha^T &= \begin{pmatrix} \nu_\alpha & \ell_\alpha & F_\alpha & \tilde{F}_\alpha \end{pmatrix}_L \sim (1, \mathbf{4}, X_{4\ell}) \end{aligned} \tag{19}$$

where the right singlets are not explicitly required because their quantization have been shown previously through their X_f^R numbers, Eqs. (14) and (18). Notice the relationship of $Q(\ell) = Q(F_\alpha)$ which implies

$$-\frac{1}{2} + \frac{\beta}{2\sqrt{3}} + \frac{\gamma}{2\sqrt{6}} + X_{4\ell} = -\frac{\beta}{\sqrt{3}} + \frac{\gamma}{2\sqrt{6}} + X_{4\ell} \tag{20}$$

and therefore $\beta = \sqrt{3}/3$. From the relationship $Q(\ell) = Q(\tilde{F}_\alpha)$, we have

$$-\frac{1}{2} + \frac{\beta}{2\sqrt{3}} + \frac{\gamma}{2\sqrt{6}} + X_{4\ell} = -\frac{3\gamma}{2\sqrt{6}} + X_{4\ell} \tag{21}$$

and replacing the value for $\beta = \sqrt{3}/3$, then we obtain $\gamma = \sqrt{6}/6$. The model concerning $\beta = \sqrt{3}/3$ and $\gamma = \sqrt{6}/6$ has been studied by Ref. 8 and it corresponds to what they called model B.

Other type of models arise when electric charges satisfy $Q(\nu_\alpha) = Q(\tilde{F}_\alpha)$, where

$$\frac{1}{2} + \frac{\beta}{2\sqrt{3}} + \frac{\gamma}{2\sqrt{6}} + X_{4\ell} = -\frac{3\gamma}{2\sqrt{6}} + X_{4\ell} \tag{22}$$

and taking $Q(\ell) = Q(F_\alpha)$, then we have

$$-\frac{1}{2} + \frac{\beta}{2\sqrt{3}} + \frac{\gamma}{2\sqrt{6}} + X_{4\ell} = -\frac{\beta}{\sqrt{3}} + \frac{\gamma}{2\sqrt{6}} + X_{4\ell} \tag{23}$$

and solving the equations, we obtain

$$\beta = \frac{\sqrt{3}}{3} \qquad \text{and} \qquad \gamma = -\frac{2\sqrt{6}}{6} \,, \tag{24}$$

this model has also been studied previously by Ponce and Sanchez[8] and it is named model F. As we can see, by fixing the parameters β and γ and using them in (4) we obtain the X numbers of the scalar sector and therefore the X_f numbers.

On the other hand, we have the model built up by Pisano and Pleitez[6] which choose the lepton sector in a different way $l_\alpha^T = \begin{pmatrix} \nu_\alpha & \ell_\alpha & \nu_\alpha^C & \ell_\alpha^C \end{pmatrix}_L$. Following the analysis shown above, we notice that the electric charge operator for the leptonic sector can be written as $Q_l = T_3 + \beta T_8 + \gamma T_{15} + X_\rho$. Using Eqs. (4), the charge operator for the leptons arise naturally and it is

$$Q_l = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} - \frac{3\beta}{2\sqrt{3}} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} - \frac{\beta}{2\sqrt{3}} - \frac{2\gamma}{\sqrt{6}} \end{pmatrix} \tag{25}$$

where it can be seen that the values for β and γ should be $-1/\sqrt{3}$ and $-2\sqrt{6}/3$, respectively. And, again electric charge is quantized. We should clarify that using the obtained values of β and γ in Eq. (4) then $X_\xi = X_\rho = 0$ and therefore they mix between them. Thus, the number of scalars is reduced to three and it is necessary to add a decuplet with $X = 0$ number in order to give masses to all fermions of the spectrum.[6] In this model, a procedure to explain electric charge quantization was proposed by Doff.[30]

Models with and without exotic electric charges in the spectrum have been studied in the literature.[6–8] Reference 8 classifies the models without exotic electric charges where they found eight different anomaly free models. Models referred as models B and F have been analyzed here and we found that they quantized the electric charge. The same is true for models A and E which have a spectrum in a representation that completely conjugate with respect to models B and F. These model A, B, E and F are three family models. On the other hand, on Ref. 8, they also found five models (C, D, G, H and I) which cancel out the anomalies using only one or two families, but these are not realistic. Finally, regarding models that include exotic electric charges, we have studied here the case of a model by Pisano and Pleitez.[6] There is also another model presented in Ref. 7 that includes right handed Majorana neutrinos in the spectrum where the electric charge quantization is obtained analogously to the one obtained in the model by Pisano and Pleitez.[6]

In summary, we have shown that the electric charge can be quantized in models based on the gauge symmetry $\mathrm{SU}(3)_c \otimes \mathrm{SU}(4)_L \otimes \mathrm{U}(1)_X$ that use three families to cancel out the chiral anomalies. We have shown this quantization using electric charge conservation, invariance under parity transformations and chiral anomaly conditions. We have shown that electric charge quantization can be obtained in models based on 341 gauge symmetry, in models including exotic particles and also in models which do not include exotic electric charges in the spectrum.

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Appendix A

In this Appendix, we explicitly show the spectrum of the models discussed here. The fermionic spectrum is according to the charge operator defined in Eq. (1) and the anomaly cancellation condition. It is worth to notice that one of the 4-plets with quarks should be in the adjoint representation different to the other two quadruplets,

$$\begin{gathered}\begin{pmatrix} d_i & u_i & J_i & \tilde{J}_i \end{pmatrix}^T_L \sim 4^* , \qquad \begin{pmatrix} d_3 & u_3 & J_3 & \tilde{J}_3 \end{pmatrix}^T_L \sim 4 , \qquad \begin{pmatrix} \nu_\alpha & \ell_\alpha & F_\alpha & \tilde{F}_\alpha \end{pmatrix}^T_L \sim 4 , \\ d_{\alpha R} \sim 1 , \qquad u_{\alpha R} \sim 1 , \qquad J_{\alpha R} \sim 1 , \\ \tilde{J}_{\alpha R} \sim 1 , \qquad \ell_{\alpha R} \sim 1 , \qquad F_{\alpha R} \sim 1 , \qquad \tilde{F}_{\alpha R} \sim 1 ,\end{gathered} \tag{A.1}$$

where J_i $(i = 1, 2)$, J_3 and F_α $(\alpha = 1, 2, 3)$ are heavy fermions.

The scalar sector is defined taking into account the spontaneously symmetry breaking pattern $\mathrm{SU}(3)_C \otimes \mathrm{SU}(4)_L \otimes \mathrm{U}(1)_X \to \mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X \to \mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \to \mathrm{SU}(3)_C \otimes \mathrm{U}(1)_Q$ and in particular the VEVs used are in Eq. (3).

Finally, following the SU(4) structure, the gauge boson fields are written as

$$\frac{1}{\sqrt{2}} \begin{pmatrix} D^0_{1\mu} & W^+_\mu & K_{1\mu} & X_\mu \\ W^-_\mu & D^0_{2\mu} & K_{2\mu} & V_\mu \\ \bar{K_1}_\mu & \bar{K_2}_\mu & D^0_{3\mu} & Y_\mu \\ \bar{X}_\mu & \bar{V}_\mu & \bar{Y}_\mu & D^0_{4\mu} \end{pmatrix} , \tag{A.2}$$

where the neutral fields are defined through

$$\begin{aligned} D^0_{1\mu} &= W^3_\mu + \frac{W^8_\mu}{\sqrt{3}} + \frac{W^{15}_\mu}{\sqrt{6}} , \\ D^0_{2\mu} &= -W^3_\mu + \frac{W^8_\mu}{\sqrt{3}} + \frac{W^{15}_\mu}{\sqrt{6}} , \\ D^0_{3\mu} &= -\frac{2W^8_\mu}{\sqrt{3}} + \frac{W^{15}_\mu}{\sqrt{6}} , \\ D^0_{4\mu} &= -\frac{3W^{15}_\mu}{\sqrt{6}} . \end{aligned} \tag{A.3}$$

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