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A note about graphical representations to capture the consumer's perception of a brand

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Abstract

Classical positioning researches use descriptive statistical methods that generate graphical displays, from a two-way table of frequencies or contingency table, in order to investigate relationships between brands and individual preferences for those brands. Most of times, the strategic plan of the company is based in such results. This paper presents a graphical methodology, based in a double-weighted correspondence analysis followed by two stage clustering, that attempts to plot the mind's perception of the customer with respect to all of the brands in a competitive market. The result of this research is aimed to be a simple but powerful tool of the marketing researcher in the strategic marketing planning stage that complements the classical approaches and expands the vision of the managing staff such that good decisions could be taken.

Keywords : *Biplot, brand equity, business strategy, clustering, correspondence analysis, positioning.*

1. Introduction

The principal output of a Correspondence Analysis (CA) is a perceptual map. This kind of approach is very useful when describing differences and association between brands by the attributes of a specified market. In marketing research it is very important to plot the existing relationships such as how the customer perceives them. However, in terms of strategy, the classical Correspondence analysis output can only answer to the following questions (Hoffman and Frenke, 1986):

- What are the similarities and differences among the brands with respect to the market attributes?

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- What are the similarities and differences among the market attributes with respect to the brands?
- What is the relationship among the brands and attributes?
- Can these relationships be represented in a joint low-dimensional space?

The four answers to the previous questions are not enough in order to make an effective strategic plan. In order to make the strategic marketing planning process to be successful, the researcher must know the answers to, not only the four previous questions, but also the following questions:

- Who are my customers? Who are the competitor's customers? Who else should be my customers?
- Where is my brand positioned relative to the other market brands? Why is my brand positioning there?
- What new products should my brand create?
- What segment of the market should I target for my brand's products?
- What is the driver attribute of the market? Is this attribute the same driver in all of the segments of the market?
- Should my brand's strategy be focused in this driver attribute?

When answering that questions, the market researcher is faced up with the construction of a strategy. If the researcher is able to know the mind's perception of the consumer in the specific market category, then he/she is able to refine tactics by

(i) focusing in the correct key drivers attributes and the correct segments of the category,
(ii) giving the consumer what they really want,
(iii) minimizing the effect of the competitors in the market,
(iv) moving forward before the rest of brands in the market in order to recruit new customer and to keep old customers for a long time.

What are the direct effects of such strategic marketing planning process? Loyalty, high fidelity, recommendation, recordation, among others. In other words, the strategy of the brand will be more effective and it will be broadened to include all 4P's of the marketing mix. That sounds great. However, is it possible to plot the mind's perception of the consumer? Is it possible to do it with just one graphic? If it was possible, the market researcher will be able to suggest a very effective strategic plan and in this plan he/she will be able to contemplate the answers to all of the previous questions. This paper gives a simple, but powerful methodology able to generate a graphic of the market category from the consumers perspective, answering the questions of the researcher. The methodology proposed in this paper is aimed to be the right hand of the market researcher when planning an effective strategy in the market-based context and it is based on the following suggestion of Ries and Trout (1990):

> *If positioning is defined as the mental angle used to enter the prospect's mind, then the role of the entire strategy must support that tactic.*

The outline of this paper is as follows: In the next section some traditional methods, that attempt to describe the behavior of the brands in the market, are examined. In section 3, the proposed graphical solution is described. This plot is result of a proposed weighted correspondence analysis following by clustering. To achieve a comparison of the proposed method, two scenarios are considered. The last part of section 3 is focused in describing the proposed methodology step by step. This section explains the good performance of the graphical method and its advantages, in terms of interpretation and capability of analysis, in order to plan an effective marketing strategy to achieve leading positioning. Some conclusions are given, and the role of marketing research regarding to the planning strategy of a brand is discussed in section 5. Finally, the appendix is devoted to elucidate some technical details that support this proposal.

2. Some traditional statistical tools

Before we exam some traditional methods in market research, we consider two scenarios. The first case is the dream of every CEO: the brand is the leader of the market, it has the 95% of the market share, all of the attributes or key drivers are being appropriated for this brand, there is no competitor in such a category, the brand is the boost of the market with the highest levels of awareness, considerations and image. This is the perfect and ideal scenario because the brand retains these levels along the segments and demographic profiles. The other case is more realistic: the brand belongs to a competitive market whit structural differences, the market share is divided in three equivalents parts. There are emerging brands that are captivating new customers, the leader of the market is different depending of the social-demographic profiles. In this case, our brand is not the leader and it has to implement a radical strategic marketing plan in order to catch up with the leader brand.

In both scenarios, suppose data were collected from 1000 consumers on four brands, and six attributes were hypothesized to describe those brands. For each brand, respondents indicate which attribute described the brand. The data generated from such a procedure is displayed in Table 1, for the ideal scenario, and in Table 2, for the realistic scenario.

Table 1

Association frequencies of attributes by brand for the ideal scenario

	V1	V2	V3	V4	V5	V6
Brand A	952	970	961	980	952	961
Brand B	10	—	29	10	19	29
Brand C	29	20	10	—	19	10
Brand D	9	10	—	10	10	—

Table 2

Association frequencies of attributes by brand for the realistic scenario

	V1	V2	V3	V4	V5	V6
Brand A	327	345	314	292	316	320
Brand B	494	505	498	541	485	476
Brand C	69	63	72	65	66	78
Brand D	110	87	116	102	133	126

The lecture of the table is as follows: In the ideal scenario, 952 persons indicated that the attribute VI described Brand A, 971 persons indicated that the attribute V2 described Brand A, and so on. The lecture is the same in both scenarios. Of course, these data were originally zeros and ones, but they have been aggregated over individuals. The reader must note the categorical constitution of the original and aggregated data.

2.1 *Joint bars*

At a first glance, the marketer researcher is tempted and aimed to plot the result of the aggregated data by means of a frequency distribution of each brand through every one of the evaluated attributes (commonly known as bar plots). This is a good approach when the market is composed by three or four brands at most. However, a mature and competitive market is composed by dozens of brands. So then, the idea of plotting the market by means of the frequency distribution approach could be a mess because, in most occasions, the manager will not be able to answer all of the questions that he/she needs to know in order to make a successful strategic plan.

Of course, the goal is to resume the results in a simple but strategic plot. Another idea is the profile plot. In the marketing context, this kind of graphic is actually composed by two plots: On the one hand, the brand by attribute plot and on the other hand, the attribute by brand plot.

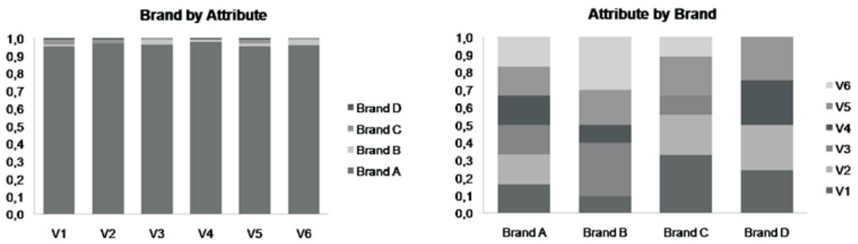

Figure 1
Brand and attribute profiles for the ideal scenario

For the ideal scenario, the profile plot is shown in Figure 1. There is no much to tell, except that Brand A is the leader brand, in a customer-based sense, and it is described by all of the attributes. Note that some other brands are not being perceived in some attributes.

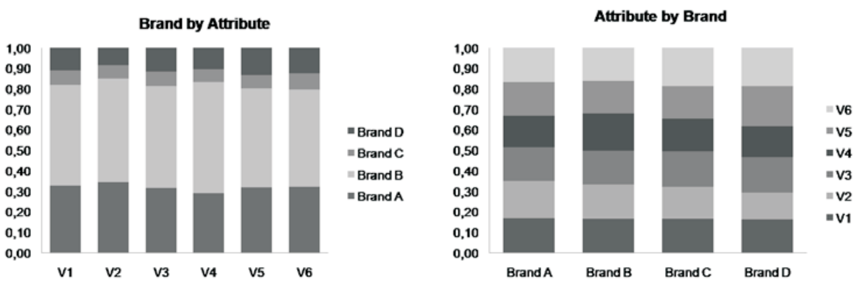

Figure 2
Brand and attribute profiles for the realistic scenario

In the realistic case, the profile plot is shown in Figure 2. In this scenario, Brand B is the leader brand followed by Brand A. However, all of the brands are being perceived as describers of every evaluated attribute. If you were the manager of Brand A and the only tool that you had were this profile plot, then you would be in a serious trouble when planning the strategic stage. There are a lot of questions that this plot is not able to answer with precision. To which attribute it would have to be focusing the propaganda? Why this attribute? How is the customer perception of my brand in the whole marketplace? Which brands are being perceived for the customer as similar?, etc. As Malhotra (2006) pointed out, this kind of approaches are very useful when obtaining a counting or relative frequency of the different values associated to brands or attributes. However, its scope is very limited in terms of plotting the consumer's mind perception of the market.

2.2 *Correspondence analysis*

Correspondence analysis (CA) is a popular data analysis method that allows to find the best representation of the association between rows and columns of the contingency table. This method enables to answer some questions that simple profile plots do not. As Withlark and Smith (2001) state, in marketing research this approach is often used in positioning and image studies where the market researcher wants to explore the relationships between brands, between attributes, and between brands and attributes.

The CA method was developed in France under the leading influence of Jean-Paul Benzecri (Benzacri, 1992). One of the pioneer approaches of this method applied in marketing research is found in Hoffman and Frenke (1986) and it could be implemented by following these steps:

(i) The CA method begins with a matrix of aggregated data, namely

$$\mathbf{X} = \begin{bmatrix} x_{11} & \cdots & x_{1J} \\ \vdots & \ddots & \vdots \\ x_{I1} & \cdots & x_{IJ} \end{bmatrix}, \tag{1}$$

where rows represent brands and columns represent attributes. The sum of all entries in the table is n, the total number of the respondents.

(ii) To construct a matrix of relative frequencies by dividing each entry by n as it is shown in the following table:

	1	2	$\cdots$	j	$\cdots$	J	row sum
1	f_{11}	f_{12}		f_{1j}		f_{1J}	$r_{1\cdot}$
2	f_{21}	f_{22}		f_{2j}		f_{2J}	$r_{2\cdot}$
$\vdots$							
i	f_{i1}	f_{i2}		f_{ij}		f_{ip}	$r_{i\cdot}$
$\vdots$							
I	f_{I1}	f_{I2}		f_{nj}		f_{np}	$r_{1\cdot}$
column sum	$c_{\cdot 1}$	$c_{\cdot 2}$		$c_{\cdot j}$		$c_{\cdot J}$	

The marginal frequencies of rows given by $\mathbf{r} = (r_{1\cdot}, r_{2\cdot}, \cdots, r_{I\cdot})'$ and the marginal frequencies of columns given by $\mathbf{c} = (c_{\cdot 1}, c_{\cdot 2}, \cdots, c_{\cdot J})'$ represent the marginal probabilities of row profiles and column profiles, respectively. These marginal frequencies can be obtained by $\mathbf{r} = \mathbf{F}\mathbf{1}_J$ and $\mathbf{c} = \mathbf{F}'\mathbf{1}_I$, where $\mathbf{F}$ is the matrix of relative frequencies, $\mathbf{1}_J$ and $\mathbf{1}_I$ are vectors of ones of dimension J and I, respectively. Note that $r_{i\cdot} = \sum_{j=1}^{J} f_{ij}$ and $c_{i\cdot} = \sum_{i=1}^{I} f_{ij}$.

(iii) To define the row profile matrix, $\mathbf{R}_{\text{profile}}$, by dividing each entry of $\mathbf{F}$ by the respective marginal row sum. Similarly, the column profile matrix, $\mathbf{C}_{\text{profile}}$, is defined. Notice that a profile can be interpreted as a conditional probability.

(iv) A row profile is a vector of dimension J, and can be represented by a point in $\mathbb{R}^J$. The set of all row-points, namely $\mathcal{H}_c$, is situated in a hyperplane $\mathcal{H}_f$ of vectors such that the sum of its components is 1.

(v) The representation of the column profiles in $\mathbb{R}^I$ is similarly defined.

(vi) The factorial analysis of row-point or column-points consists in a succession of orthogonal directions, where the inertia‡ is maximized.

(vii) The factorial map is determined by two factors of the rows or the columns and provides approximate images of $\mathcal{H}_f$ and $\mathcal{H}_c$. In these maps, the distance between two points can be interpreted as the similarity of the respective profiles. The origin of the axes is considered to be the average profile.

(viii) The relations of transition represent the results of the factorial analysis of the rows or columns in terms of the columns or rows.

(ix) Once the transitions are found, the interpretation of the factorial maps representing $\mathcal{H}_f$ and $\mathcal{H}_c$ must be made jointly in $\mathbb{R}^2$. The interpretation of this simultaneous representation is very useful for drawing conclusions and is known as the so-called perceptual map.

Row-points that are close together indicate brands that have similar profiles based on a conditional distribution across the attributes of the market, column-points that are close together indicate attributes with similar profiles down the evaluated brands. So much has been discussed

‡The inertia is defined to be the overall spatial variation in each set of points. It is defined as the weighted sum of squared distances to the origin of the displayed row or column profiles. An optimal representation should retain a total inertia near close to unity.

about the interpretation of distance between brands and attributes. Carroll, Green and Schaffer (1986), Greenacre (1989), and Carroll, Green and Schaffer (1989) provide an expository account of distances in correspondence analysis. The authors of this paper are motivated by the simple idea exposed by Johnson and Wichern (2006) where row-points that are close to column-points represent combinations that occur more frequently than would be expected from an independent model in which the row categories are unrelated to the column categories.

For example, the interpretation of Figure 3, in the ideal scenario, would be as follows: Notice that the points of Brand B, Brand C and Brand D are quite distant from each other and separated from the remaining Brand A, and from the evaluated attributes. This indicates that Brand A tends to be associated, almost exclusively, with all of the attributes. Attributes V3 and V6 are perceived similarly, because they are sharing almost the same factorial coordinates. Together, the two dimensions account for $67\% + 29\% = 96\%$ of the total inertia. This plot is considered the best two dimensional representation of the multidimensional scatter of row-points and column-points. Of course, Figure 3 shows that Brand A is the leader brand.

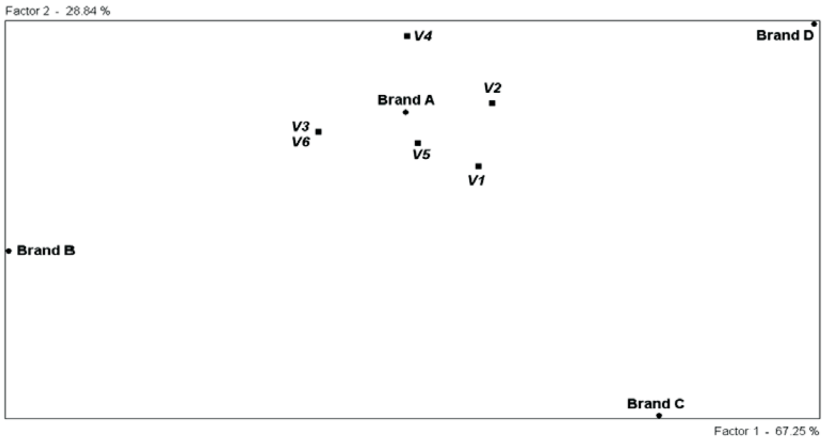

Figure 3

Correspondence analysis for the ideal scenario

However, the same methodology applied to data in Table 2 could guide to misleading interpretations. In the more realistic scenario, if a naive manager had to develop a strategic marketing plan only by examining Figure 4. He/she would find great issues by interpreting this plot. By the traditional interpretation, Brand C tends to be associated with

Attributes V3, V5 and V6. Notice that this brand is near close to the center of the plot, which commonly indicates that this brand is the average brand of the market. This way, Brand B could be perceived as a poor brand, in terms that there is no attribute strongly associated with this brand. Brand A and Brand D are quite distant from almost all of the attributes. On the other hand, Figure 4 explains $65\% + 30\% = 95\%$ of the total inertia and Attributes V5 and V6, such as VI and V3, are perceived similarly.

If the supposed manager had no access to the frequency distributions shown in Table 2, then he/she may misrepresent marketplace perceptions. Of course, this situation is so unrealistic, but the point is that managers want a tool that allows them to take right decisions when answering all of the questions formulated in the background of this paper and as simple as possible. Indeed, the purpose of this paper is to proportionate this kind of tool.

3. Graphical solution

We have shown that misleading conclusions could be taken by focusing just in the plot of the CA. However, this kind of approach could become a powerful tool that considers the whole scenario of the marketplace. Just think about yourself and the way you perceive a market. Is there only such points and squares? The idea of the CA applied to the marketing research would be so wonderful only and only if the researcher considered all of the associations and perceptions of the customer in the changeling market. Of course, this attempt to capture the whole marketplace could be simply impossible because, as Zaltman (2003) declares, customers do not think in a linear form; indeed, they do what they do and they choose what they choose guided by emotional and unconscious decisions.

So, the solution proposed in this paper is a very partial one. However, it goes one step further through the quantitative approach in order to understand the perception of the .market from the customer point of view. In this proposal we have taken in account the following items:

- The importance of the brands and their contribution to the marketplace.
- The importance of the attributes and their contribution to the marketplace.
- The relationship between brands and attributes and how strong this association is.
- The mental segmentation process of the mind's customer.

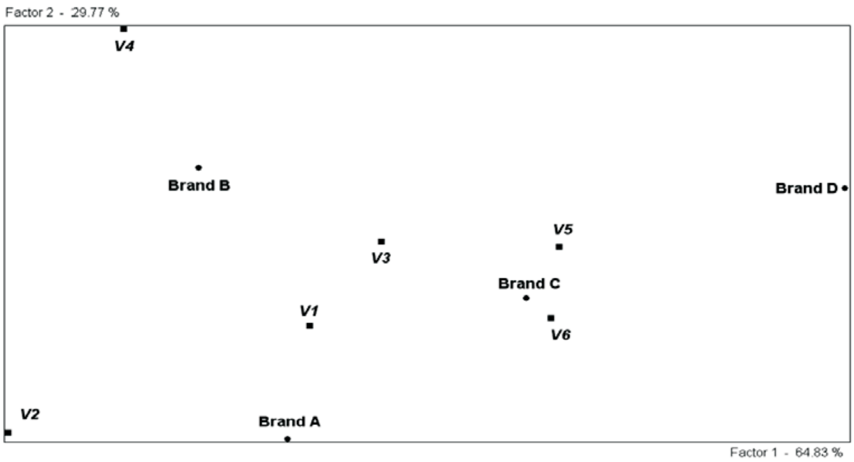

Figure 4
Correspondence analysis for the realistic scenario

The simplicity in the lecture and easiness of interpretation leads this approach, which is based in a proposed weighted CA followed by clustering over the resulting factorial axes.

3.1 *Weighted correspondence analysis*

When performing a correspondence analysis of aggregated data, the general assumption is that a product of a mature market is composed by attributes and that a market is composed by brands. In the following lines we proceed to explain the proposed approach retaining these assumptions of the correspondence analysis.

Under the former suppositions, Kano, Seraku, Takahashi and Tsuji (1984) have developed a great theory about customer satisfaction. We believe that a perceptual map must show how different the evaluated attributes are and how they are impacting the mind's customer. For example, in Malhotra (2006, chapter 21) there is an empirical example about sport shoes. In this research, was found that the key attribute of this category was the price, followed by the material of the sole and the material of the shoe. The same line of thinking could be followed in any category of any market: There are attributes more important than others and the customer knows it.

On the other side, there are brands: there are leader brands and developing brands. This difference also must be displayed in the plot, because the customer thinks in this way. However, the importance of any brand in the market cannot be measured by some indicators such as the market share. Note that when a customer is going to purchase a product, he/she is not making calculations about this kind of information reserved only to brand managers. The question that arises is: How could be measured the importance of an attribute and the impact of a brand in a mature market?

On one hand, about the importance of attributes, so much have been discussed in these topics, and there are statistical and nonstatistical techniques used to solve this issue, such as direct question to the customer, discrete choice, conjoint analysis, multidimensional scaling, factor analysis and linear models. Actually, the how is out of the scope of this paper. The point is that we suppose that auxiliary information, about the importance of every evaluated attribute, is available. For example, following the empirical exercise of Malhotra (2006), it was found that the price was the key driver attribute with a 50% of importance; the material of the sole had a 28% of importance and the material of the shoe had a 22% of importance.

On the other hand, about the impact of brands, we suppose that a brand has more impact in the mind's perception of a customer if it is associated with the important attributes; i.e., actually, it is more impacting a brand that is strongly associated with a couple of the most important attributes, than a brand that is surrounded by attributes that are not important from the customer point of view. Notice that impact is related with market share but it is strongly correlated with brand equity.

Based on the aforementioned, we propose to modify the classical CA as follows:

(i) To weight every column of the aggregated data matrix by its relative importance given by the auxiliary information
(ii) To apply a traditional CA to this weighted matrix.
(iii) Finally, to weight the sizes of the row-points and column-points by their relative weighting.

This way, the sum of each column will be equal to the relative importance of the attribute and the sum of each row will be equal to the relative impact of the brand in the market. Notice that this kind of approach gives more impact to the brands that are well-scored in the important attributes and gives less impact to the brands that are bad-scored in the important attributes. The theoretical details of the procedure and how to construct this approach, namely double weighted correspondence analysis plot (DWCAP), are presented in the appendix. We recall that this paper is not focused in the statistical interpretation of factorial axes, but in characterizing brands by attributes in a two-dimensional space.

Notice that in the classical CA plot, all the row-points and column-points have the same size, so there is no way that we can decide which brand is the most impacting or which attribute is the most important from the mind's customer point of view. In the proposed DWCAP, we use the auxiliary information about the importance of attributes to weight the size of the column-points in the plot, such that the larger the value of auxiliary information for the j-th attribute, p_j, the larger the size of the column-point in the plot and more important is the corresponding attribute regarding other attributes. Similarly, we use the row sums of the relative frequencies matrix (see expression (3) below in appendix) to weight the size of the row-points, so larger the size of the row-points, we conclude that more important is the corresponding brand regarding other brands. For instance, we present the results obtained when we apply the DWCAP to the two scenarios presented in section 2: the ideal scenario presented in the Table 1 and the realistic scenario in the Table 2, and we show that this plot is a comprehensive tool that does consider into account not only the relationship between brands or attributes but also the strength of these relationships and the level of impact or importance of every brand or attribute, just the way the consumer does. First, consider the ideal scenario, we obtain the relative frequencies matrix by dividing each entry of Table 1 by 1000, the total number of the respondents. Now, suppose that auxiliary information about the importance of attributes is available from a confident source (some previous research or maybe direct question to the respondents). For example, suppose that the importance of Attribute VI is 70%, of Attribute V2 is 10%, of Attribute V3 is 5%, of Attribute V4 is 5%, of Attribute V5 is 7% and finally of attribute V6 is 3%. Note that the weight of attribute VI is dominant among all of the remaining attributes. With this auxiliary information we proceed to calculate the weighted relative frequencies (see expression (4) below in appendix) by multiplying the relative frequencies matrix by the auxiliary information. The resulting weighted relative frequencies table is displayed in Table 3.

Analogously, we obtain the weighted relative frequencies table, for the realistic scenario, supposing that auxiliary information is available in such way that the importance of Attribute VI is 26%, of Attribute V2 is

14%, of Attribute V3 is 22%, of Attribute V4 is 16%, of Attribute V5 is 11% and, of Attribute V6 is 11%. Note that in this scenario, the weights of the attributes are more balanced. Table 4 presents the relative frequencies table for this realistic scenario.

Table 3

Weighted relative frequencies of attributes by brand for the ideal scenario

	V1	V2	V3	V4	V5	V6
Brand A	0.6664	0.0970	0.0481	0.0490	0.0666	0.0288
Brand B	0.0007	—	0.0014	0.0005	0.0013	0.0009
Brand C	0.0203	0.0020	0.0005	—	0.0013	0.0003
Brand D	0.0063	0.0010	—	0.0005	0.0007	—

Table 4

Weighted relative frequencies of attributes by brand for the realistic scenario

	V1	V2	V3	V4	V5	V6
Brand A	0.0850	0.0483	0.0691	0.0467	0.0347	0.0352
Brand B	0.1285	0.0707	0.1096	0.0866	0.0534	0.0524
Brand C	0.0179	0.0088	0.0158	0.0104	0.0073	0.0086
Brand D	0.0286	0.0122	0.0255	0.0163	0.0146	0.0138

The resulting DWCAP of the ideal scenario is presented in the Figure 5 while the DWCAP of the realistic scenario is presented in Figure 6.

Notice that the size of every brand and attribute is weighted by its relative impact and importance, respectively. The interpretation of Figure 5 follows the same outline that the interpretation of Figure 3. Of course, this plot still shows that the Brand A is the leading brand, so far and it is strongly associated with the most important attribute.

The interpretation of Figure 6 is quite different than the interpretation of Figure 4. This plot shows Brand B as the leading brand and Brand C as a developing brand; indeed this brand is perceived as the poorest brand of the market. The size of the circle of Brand B is larger than the others, it indicates that Brand B has the highest scores in all of the evaluated attributes. The same interpretation should be stated for Brand C; the size of the circle of this brand is the smallest indicating that the scores for this brand are the lowest in each attribute. Notice that even though this brand is surrounded by three attributes: V3, V5 and V6, it is not true that Brand C is well-perceived by the customer. The attribute VI is perceived as the more important attribute in the market, and Brand A is closer to this attribute than any other brand. It makes sense when realizing that attribute VI is the best-scored attribute of this brand (See Table 4). This plot shows the whole marketplace and it has into account many things that classical CA does not. Note that it is hard to make a mistake when interpreting this plot because of the size of the brands and attributes. All of the remaining stuff as the interpretation of the axes and the inertia could be explained in the same way as in a classical CA.

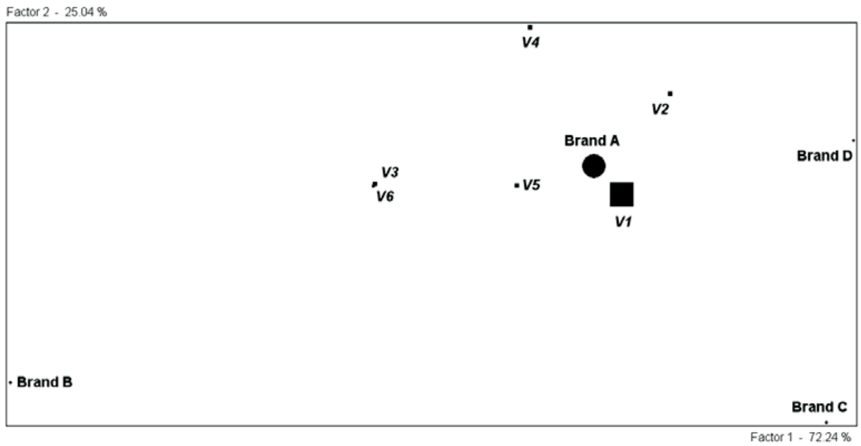

Figure 5
Double weighted correspondence analysis for the ideal scenario

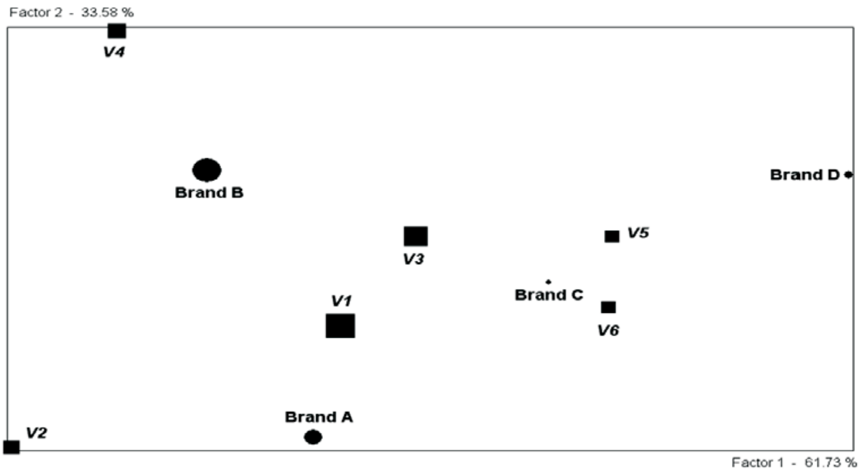

Figure 6
Double weighted correspondence analysis for the realistic scenario

3.2 *Segmentation*

Although the previous methodology gives account of a wider perspective of the marketplace, it does not consider a process that the customer does unconsciously: market segmentation. Traditionally, the market researcher is interested in partitioning the market with the purpose of selecting one or more market segments that the company can target through the specific marketing mixes that adapt to particular marketing needs. However, we suppose that the customer also performs a mental partition of the market and he/she places him/herself in a specific segment in order to purchase a product or brand.

The segmentation technique depends largely of the data available (continuous or categorical variables), and the types of dependence observed. Among the most common segmentation techniques used are factor analysis, cluster analysis, discriminant analysis and generalized linear models.

3.2.1 Two stage clustering

This kind of partition could be achieved through the coordinates of the induced factorial axes of the proposed DWCAP. Note that as a result of the application of this methodology, two factorial axes are generated[§]. We propose to perform a two stage clustering of brands in order to segment the marketplace and to emulate the segmentation process of the customer.

The first stage of the segmentation is based in a hierarchical algorithm using the centroid method. Note that the coordinates, for every evaluated brand, of the induced factorial'axes namely, $\mathbf{a}_1$ and $\mathbf{a}_2$, are in $\mathbb{R}^I$, thereby it is pertinent to implement this kind of method because of the continuous nature of this axes. The principal output of this method is a dendrogram, defined as a graphical display of the agglomeration processes, with which the market researcher evaluates every linkage and makes a decision about the suitable number of cluster pertinent in the marketplace. This decision must be based in the seniority, expertise and knowledge of the market researcher about the whole marketplace.

When a number of cluster is defined, then the second stage of the clustering approach begins. For instance, it is recommended to implement a mobile center algorithm that enables to improve the classification of every brand. Once again, the authors emphasize that the interpretation

[§]In fact, we have chosen the first two factorial axes induced by the eigenvectors of the two largest eigenvalues of the Singular Value Decomposition of the centered relative frequency matrix (see expression (9) below in the appendix).

of every cluster must make sense from a statistical and marketing point of view¶. However it is stressed that the interpretation of the cluster implies a comprehensive examination of the ultimate centers, which represent the average values of the brands contained in every cluster.

When the clusters – made up of brands associated with attributes – are defined, the next step in the proposed graphical solution of this paper is to draw them into the DWCAP output by means of ellipses. This way, the manager will have a wider vision of the role of every brand in the market. Even though the clusters were based in brands, we must to recognize that some brands are more or less associated, in the sense of chi-squared test, with attributes. Based on the aforementioned, the interpretation of these ellipses must be done carefully. For example, if a brand is outside of a specified cluster, it does not mean that this brand is not associated with the attributes inside the cluster. On the contrary, it means that all of the brand are associated with every evaluated attribute, but the brands belonging this specified cluster are more associated with the attributes inside the cluster.

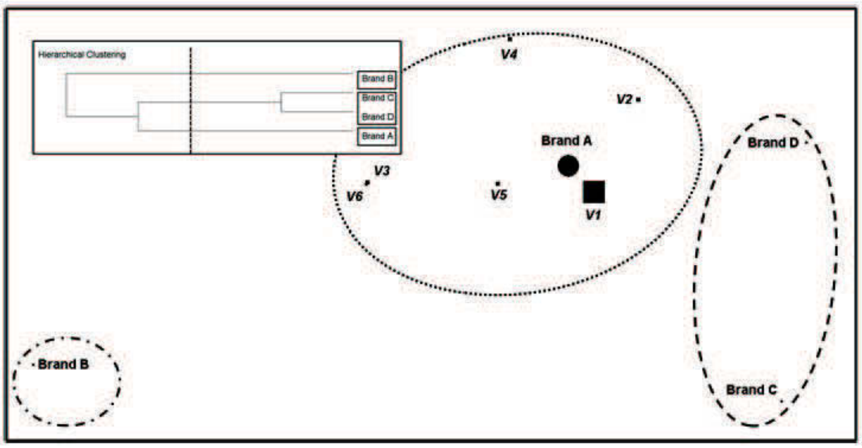

Figure 7
Proposed graphical approach for the ideal scenario

When the proposed approach is applied to data in Table 1, the output of the DWCAP with the incorporation of clusters is displayed in Figure 7. Of course, essentially it is the same graphic, but the ellipses

¶The interpretation of the statistical methods, and their outputs, used in the clustering approach is out of the scope of this research. However, it is supposed that the marketing researcher has the capability of interpretation not only in the statistical sense but also, and most important, in the marketing sense.

give a comprehensive understanding of this ideal market[‖]. There are no surprises in the findings: Brand A is the leader brand of the market. There are no other brand like this one. It boosts the behavior of the market because all of the attributes are strongly associated with it. However, there are two brands, Brand C and Brand D, appealing with the same general perception. Brand B is not strongly associated to any other brand.

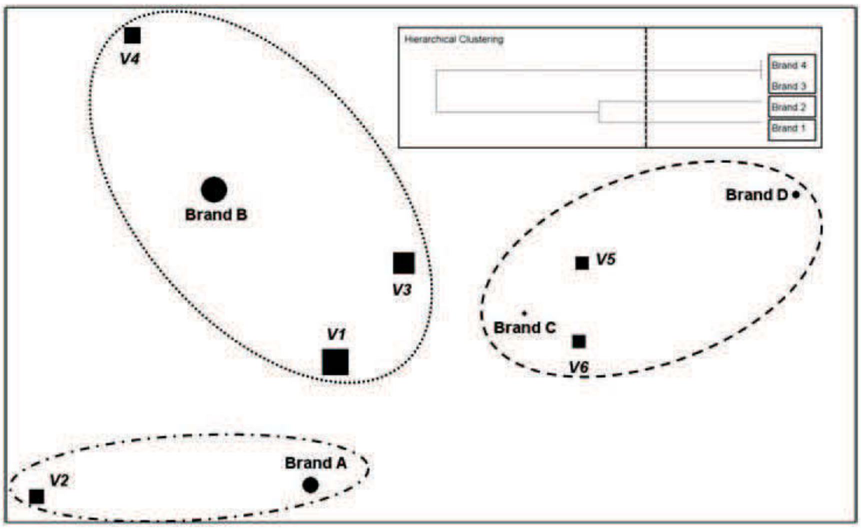

Figure 8

Proposed graphical approach for the realistic scenario

The realistic scenario presents quite different characteristics from the other scenario. The first cluster contains only one element: Brand B, which is the leading brand in the market and has regular associations[**] with attributes VI, V3 and V4. Notice that this cluster contains the leading brand alone and the three most important attributes of the market. Another cluster is composed by Brand A and the attribute V2. The last cluster is composed by Brand C and Brand D sharing attributes V5 and V6.

If a strategic marketing plan should be addressed by the managers of any brand, this plot would become a strategic chart of the company, because it displays the whole perception of the average customer and answers the question asked in the introduction of this paper and it informs

[‖]Notice that ellipses contains strictly the brands in specified clusters. However the expertise of the researcher is needed in order to decide which attribute must be included in the ellipses.

[**]The authors recall that ellipses should be build by experts of the particular market.

about the position of the brand, in terms of clustering, in the market. This way, the manager is able to establish a better strategy.

4. Concluding remarks

The proposed approach could be classified as a perceptual map. It provides a comprehensive display of the market just as the average customer perceives it. We have found that some auxiliary information about the importance of every attribute can be attached to the classical CA in order to improve the output, in terms of strategic analysis. Since that the principal output of the proposed approach provides a direct visualization and interpretation about the impact of the brand and the importance of every evaluated attribute, then it is recommended to be used when designing any marketing strategic plan in order to improve the positioning and equity of the brand. Our statement is that the customer makes an unconscious segmentation of the market when selecting, purchasing and paying for a product. If the manager is able to know where his/her brand is and which brands are sharing a similar perception in the customer mind, then he/she will be able to improve dramatically the position of the brand.

We have supposed that data has been aggregated. However this approach could be used in several different scenarios. For example, the columns of the data matrix could be age groups, social-economic membership, or any other demography characteristic of interest. Also, the entries of the data matrix could indicate use frequency or overall ratings of every evaluated brand by attribute. This approach is aimed to be a useful resource when constructing brand image index, brand equity index and share of quality index. Of course, one plot is not enough to display the whole marketplace in order to take good decisions. We strongly encourage to the users of this approach to have maps by social-demographic profiles, temporal tracking, location and in the case of services maps by every specific section, moment and aspect that the customer have to afford in order to reach the entire service.

A. Technical details

A.I *DWCAP*

In this section, we present the theoretical development about how to construct the proposed approach. The analysis begins with the aggregated data matrix, $\mathbf{X}$, given by

$$\mathbf{X} = \begin{bmatrix} x_{11} & \cdots & x_{1J} \\ \vdots & \ddots & \vdots \\ x_{I1} & \cdots & x_{IJ} \end{bmatrix}. \tag{2}$$

Recalling the practical context, the sum of every column is the same, say m. We transform the matrix $\mathbf{X}$ by dividing each entry by the column sum m. And we obtain the relative frequency matrix $\mathbf{F}$ given by

$$\mathbf{F} = \begin{bmatrix} f_{11} & \cdots & f_{1J} \\ \vdots & \ddots & \vdots \\ f_{I1} & \cdots & f_{IJ} \end{bmatrix}. \tag{3}$$

Note that in a classical correspondence analysis, the sum of all entries of the frequency matrix is 1, but in this case, this sum is equal to J, and the column sums are equal to 1.

Additionally, auxiliary information, about the importance of every evaluated attribute, is available and it is given by $\mathbf{p} = (p_1, \cdots, p_J)'$, where $\sum_{j=1}^{J} p_j = 1$. In order to incorporate this information in the analysis, we must multiply each entry of the matrix $\mathbf{F}$ by the weight of the corresponding column in the vector p, that is, we compute the matricial product, $\mathbf{FP}$ where $\mathbf{P} = \text{diag}(\mathbf{p})$ and we found that the resulting matrix is given by

$$\mathbf{FP} = \begin{bmatrix} f_{11}p_1 & \cdots & f_{1J}P_J \\ \vdots & \ddots & \vdots \\ f_{I1}p_1 & \cdots & f_{IJ}p_J \end{bmatrix}. \tag{4}$$

Note that the sum of the j-th column of **FP** is equal to p_j for $j = 1, \cdots, J$, and the sum of all of the entries of the matrix **FP** is 1.

At this point, we are ready to apply the classical correspondence analysis to the matrix **FP**.

First we center the matrix **FP** by subtracting the row total times the column total for each entry so that in the resulting matrix, the row total and the column total are all zero. For that, note that vector of the row totals can be computed by $\mathbf{r} = \mathbf{FP1}_J$, where $\mathbf{1}_J$ is the vector of 1s of length J, and on the other side, the column totals $\mathbf{c}$ are the same as the auxiliary information $\mathbf{p}$. So we obtain the centered matrix $\mathbf{FP} - \mathbf{rc}' = \mathbf{FP} - \mathbf{FP1}_J\mathbf{p}' = \mathbf{FP}(\mathbf{I} - \mathbf{1}_J\mathbf{p}')$.

Then we standardize the previous matrix by dividing each entry by the square root of the corresponding row total times column total; that is, we compute the matrix

$$\mathbf{B} = \mathbf{R}^{-1/2}\mathbf{FP}(\mathbf{I} - \mathbf{1}_J\mathbf{p}')\mathbf{P}^{-1/2}, \tag{5}$$

where $\mathbf{R} = \mathrm{diag}(\mathbf{r})$.

The next step is to find the singular value decomposition of $\mathbf{B}$ given by

$$\mathbf{B} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{V}' \tag{6}$$

where U and V are orthogonal matrices of dimension $I \times I$ and $J \times J$, respectively, and Λ is a partitioned diagonal matrix of dimension $I \times J$.

Then based on (6), we can obtain the singular value decomposition of $\mathbf{FP}(\mathbf{I} - \mathbf{1}_J, \mathbf{p}')$ as follows:

$$\begin{aligned} \mathbf{FP}(\mathbf{I} - \mathbf{1}_J\mathbf{p}') &= \mathbf{R}^{1/2}\mathbf{BP}^{1/2} & (7)\\ &= (\mathbf{R}^{1/2}\mathbf{U})\boldsymbol{\Lambda}(\mathbf{V}'\mathbf{P}^{1/2}) & (8)\\ &= \widetilde{\mathbf{U}}\boldsymbol{\Lambda}\widetilde{\mathbf{V}}'. & (9) \end{aligned}$$

Where the columns of $\widetilde{\mathbf{V}}$ and $\widetilde{\mathbf{U}}$ define the coordinate axes for the row and column profiles of **FP**, respectively. Additionally we can obtain the coordinates of the row profiles as

$$\mathbf{Y}_r = \mathbf{R}^{-1}\widetilde{\mathbf{U}}\boldsymbol{\Lambda}, \tag{10}$$

and the coordinates of the column profiles as

$$\mathbf{Y}_c = \mathbf{P}^{-1}\widetilde{\mathbf{V}}\boldsymbol{\Lambda}. \tag{11}$$

Using the former expressions, we can plot the row-points and the column-points in a two-dimensional space. In order to decide which dimensions are optimal for plotting the points, we calculate the inertia of each dimension given by the following expression:

$$\mathrm{inertia}_r = \lambda_r^2 \tag{12}$$

with $r = 1, \cdots, k$, where $k = \mathrm{rank}(\mathbf{FP}(\mathbf{I} - \mathbf{1}_J\mathbf{p}'))$, and $\lambda_1 \geq \lambda_2 \leq \cdots \lambda_k$ are non-zero diagonal values of $\boldsymbol{\Lambda}$. By taking into account the inertia of each dimension, we can plot the row-points and column-points in a optimal two-dimensional space if the inertia of the two first axes exceed 80%. In the plot of the classical CA, all the row-points or column-points have the same size, so there is no way that we can decide which brand or which attribute is the most important. In the proposed DWCAP, we use the auxiliary information about the attributes to change the size of the column-points in the plot, such that the larger the auxiliary information p_j, the larger the size of the column-point in the plot and consequently more important is the corresponding attribute regarding to the remaining attributes. Similarly, we use the row sums to change the size of the row-points in the plot, in such way that the large the size of the row-points, more important is the corresponding brand regarding to the other brands in the market. The following result supports this proposal.

Result 1. *Given the previous theoretical background, the following relations hold:*

(a) *the weighted average of the row profiles are equal to the vector of auxiliary information* $\mathbf{p}$*, and*

(b) *the weighted average of the column profiles are equal to the row sums of the matrix* $\mathbf{FP}$*, that is* $\mathbf{r}$*.*

Proof. First consider the matrix of row profiles given by:

$$\mathbf{R}_{\text{profile}} = \mathbf{R}^{-1}\mathbf{FP} \tag{13}$$

and the matrix of column profiles, in our case, given by:

$$\mathbf{C}_{\text{profile}} = \mathbf{P}^{-1}(\mathbf{FP})' = \mathbf{F}'. \tag{14}$$

Now, we can define the weighted average of the row profiles as

$$\mathbf{R}'_{\text{profile}}r = \mathbf{PF}'\mathbf{R}^{-1}\mathbf{FP1}_J\,. \tag{15}$$

But

$$\mathbf{R}^{-1}\mathbf{FP} = \begin{bmatrix} \dfrac{f_{11}p_1}{\sum\limits_j f_{1j}p_j} & \cdots & \dfrac{f_{1J}p_J}{\sum\limits_j f_{1j}p_j} \\ \vdots & \ddots & \vdots \\ \dfrac{f_{I1}p_1}{\sum\limits_j f_{I_j}p_j} & \cdots & \dfrac{f_{IJ}p_J}{\sum\limits_j f_{I_j}p_j} \end{bmatrix}, \tag{16}$$

so $\mathbf{R}^{-1}\mathbf{FP1}_J = \mathbf{1}_I$, replacing in (15), we obtain that the weighted average of the row profiles is

$$\mathbf{PF}'\mathbf{1}_I = \begin{bmatrix} f_{11}p_1 & \cdots & f_{I1}p_1 \\ \vdots & \ddots & \vdots \\ f_{1J}p_J & \cdots & f_{IJ}p_J \end{bmatrix} \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} = \begin{bmatrix} p_1 \\ \vdots \\ p_J \end{bmatrix} = \mathbf{p}. \tag{17}$$

In conclusion, the weighted average of the row profiles is equal to the auxiliary information about the columns.

Similarly we compute the weighted average of the column profiles as

$$\begin{aligned}
\mathbf{C}'_{\text{profile}}\mathbf{c} &= \mathbf{C}'_{\text{profile}}\mathbf{p} && (18)\\
&= \mathbf{C}'_{\text{profile}}(\mathbf{FP})'\mathbf{1}_I && (19)\\
&= \mathbf{FPF}'\mathbf{1}_I && (20)\\
&= \mathbf{F}\begin{pmatrix} p_1 \\ \vdots \\ p_J \end{pmatrix} \quad \text{(see equation (17))} && (21)\\
&= \begin{bmatrix} f_{11}p_1 + \cdots + f_{1J}p_J \\ \vdots \\ f_{I1}p_1 + \cdots f_{IJ}p_J \end{bmatrix} && (22)\\
&= \mathbf{r}. && (23)
\end{aligned}$$

In conclusion, the weighted average of the column profiles are equal to the row sums of the matrix **FP**. □

Note that $p_j = \sum_i = 1^I f_{ij}p_j$ is defined to be the relative weighting of the j-th attribute and similarly, $r_i = \sum_j = 1^J f_{ij}p_j$ is defined to be the relative weighting of the i-th brand, with $j = 1, \cdots, J$ and $i = 1, \cdots, I$.

A.2 *Clustering approach*

Based in the DWCAP output, we can proceed with the classification stage of the brands. The first step is to find a factorial map for the row-points, that is, a factorial map for the brands. In equation (9), the columns of the matrix $\widetilde{\mathbf{V}}$ defines the coordinate axes for the row-points, and the columns of the matrix $\mathbf{Y}_r$ contains the coordinates of the row-points. We use the first two columns such that the corresponding inertias exceed 80%, we denote the matrix containing these columns as $\mathbf{A}_2$, and denote its two columns as $\mathbf{a}_1$ and $\mathbf{a}_2$. Note that we can assume that these two columns take continuous values, and belong to $\mathbb{R}^I$.

The dendrogram or hierarchical tree is a graphical representation of the result of a clustering analysis. We construct the dendrogram in order to choose a suitable number of clusters grouping brands. The corresponding steps are as follows:

(i) Initially, we define I clusters, where each brand is defined as one cluster, and then we group the two closest brands in order to define a new cluster.

(ii) To calculate the distance between this new cluster and each of the remaining $I-2$ clusters. The way to calculate these distances is using the centroid distance, given that the coordinates of the brands are continuous-type. The distance between a cluster C and a new cluster induced by two old clusters: A and B with n_A and n_B brands, respectively, is defined as follows:

$$d^2(C, AB) = \frac{n_A}{n_A + n_B} d^2_{CA} + \frac{n_B}{n_A + n_B} d^2_{CB} - \frac{n_A n_B}{(n_+ n_B)^2} d^2_{AB}, \quad (24)$$

where $d(AB)$ denotes the Euclidean distance between the clusters A and B.

(iii) To repeat the algorithm until all of the brands are grouped in a whole cluster. The dendrogram indicates the clustering order so we can know which brands belong to each cluster.

We can improve the classification by using the method of mobile centers. First of all, we must choose a suitable number of clusters, namely K. The way to implement this method is as follows:

(i) To choose the K farthest brands as the centers of the K clusters.

(ii) For each of the remaining $I-K$ brands, to calculate its distance from the K centers, and attach it to the closest cluster, then recalculate the center of the corresponding cluster.

(iii) To calculate the sum of squares between groups to check out whether reassigning some brands the sum of squares decreases.

(iv) To determine the classification that minimizes the sum of squares between groups.

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