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DYNAMIC ANALYSIS AND SIMULATION OF COMPUTED TORQUE CONTROL OF A PARALLEL ROBOT 3SPS-1U

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***Abstract*— The parallel robots have entered the industry with the passage of time, being the mathematical model very important to the development of control strategies allowing obtain a better performance. So an analysis dynamic to find the parameters of the dynamic equation characteristic is made using the method of Euler - Lagrange, and expressions generals are found, so these can be used to other parallel robots with a major number of degrees of freedom. The simulation of the dynamic is made in Matlab, then a computed torque control is proposed and simulated using Matlab - Simulink. In this way, the performance of the controller can be analyzed.**

I. Introduction

Given the growing need to reduce time, avoid accidents, increase production and automate, robotics has ventured into the industry every day [1], being very common today to find robotic manipulators of any kind in various companies. For this reason, research centers and universities delve into issues related to this area, from their mathematical model, work space analysis, singularities and especially in the development and implementation of control laws that fulfill a specific task [2].

The robots that have been mostly studied are the serial ones, however, in the last years the parallel robots are being used in the industry [3] [4], thanks to their advantages with respect to the serial manipulators, such as, greater rigidity in their structure, greater precision , ability to reach high speeds and withstand heavy loads [5].

For the design of a parallel robot that can faithfully reproduce the dynamics, it is essential to make mathematical models of kinematics [6] [7] and kinetics [8] [9], that contemplate all elements of the system and develop a controller that meets the requirements of the same, taking into account the uncertainties present.

In the present article the dynamic analysis of a parallel robot 3SPS - 1U is made using the method of Euler - Lagrange, since this is a of the most common way to find every one of the terms of characteristic equation of this robot kind [10]. Additionally this method is easier of implement with respect to other methods, for example the method of Newton - Euler. After the simulation is done with the parameters physics owns of the parallel robot. Finally the strategy of computed torque control is proposed and its development mathematical and simulation is shown.

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II. Dynamic Analysis

The dynamic analysis of a robot, whether serial or parallel, consists in the study of the movement considering the torques and the forces that produce it [11] [12]. In the same way as in the kinematics, the dynamics are also divided in inverse and direct. In the first, given the trajectory, velocities and accelerations of the mobile platform, the forces or torque of the actuated joints must be determined. In the case of the direct the opposite occurs, the forces and torque of the actuated joints are known and the trajectory, velocities and accelerations of the final effector must be calculated [13]. On the other hand, there are classic methods for the calculation of dynamics, some of them are the formulation of Euler - Lagrange, Newton - Euler and the principle of virtual works [14].

In the method of Euler - Lagrange the goal is find an expression that allows to find the force that is exerted on the rod to carry out the movement and to describe a determined trajectory.

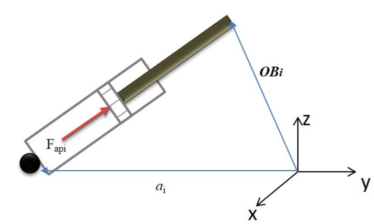


Fig. 1: Force exerted by the rod

According to the Figure 1, the vector OBi is denominated as q_{pi} for the approach of the equations, since the Lagrange expression for a dynamic system is described by

$$[F_i] = \frac{d}{dt}\left(\frac{\partial T}{\partial [\dot{q_{pi}}]}\right) - \frac{\partial T}{\partial [q_{pi}]} \tag{1}$$

where $[F_i]$ is the matrix of generalized forces that act in the system, T represents the kinetic energy and $[q_{pi}]$corresponds to the coordinate vector.

Now, the equation of energy for an actuator is going to be made, taking into account that the three actuators of the platform are identical. The actuator can be considered as a rigid body, taking into account the linear kinetic energy present, due to the axial movement made by the actuator rod. The energy equation used is that formulated in the research of Gou and Li [15] as follow,

$$T_i = \frac{1}{2} v_{vi}^T m_v v_{vi} + \frac{1}{2} \omega_{Li}^T (I_v + I_c)\, \omega_{Li} \tag{2}$$

In this, T_i represents the actuator kinetics energy, $\frac{1}{2} v_{vi}^T m_v v_{vi}$ corresponds to the translational kinetic energy of the rod and $\frac{1}{2} \omega_{Li}^T (I_v + I_c)\, \omega_{Li}$ is the rotational kinetic energy of the complete actuator. The terms $I_v + I_c$ are components of the actuator inertia matrix. In order to develop the equation, it is necessary to know the parameters of actuator speeds. For the angular velocity, considering that the analysis of the inverse kinematics was already done, we have the length of each actuator and placing an axial unit vector to it as shown in the Figure 2, we have,

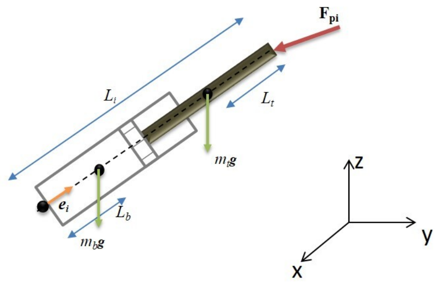


Fig. 2: Actuator Analysis

$$[\omega_{Li}] = \frac{[e_i] \times [\dot{q_{pi}}]}{L_i} \tag{3}$$

so $\dot{q_{pi}}$, is giving to,

$$[\dot{q_{pi}}] = [\dot{O}P] + [\omega] \times [R][PB_i]$$

where ω represents the angular velocity of the mobile platform. That matrix is expressed as,

$$[\dot{q_{pi}}] = \begin{bmatrix} [I] & [R][\hat{PB_i}]^T [R]^T \end{bmatrix} \begin{bmatrix} [\dot{O}P] \\ [\omega] \end{bmatrix} \tag{4}$$

So in the equation (4), the vector $\begin{bmatrix} [\dot{O}P] & [\omega] \end{bmatrix}^T$ represents the generalized coordinates and the term $[\hat{PB_i}]$ is an antisymmetric matrix, described in a general way as,

$$[\hat{PB}] = \begin{bmatrix} PB_x \\ PB_y \\ PB_z \end{bmatrix}$$

$$[\hat{PB}] = \begin{bmatrix} 0 & -PB_z & PB_y \\ PB_z & 0 & -PB_x \\ -PB_y & -PB_x & 0 \end{bmatrix} \tag{5}$$

Now, the velocity vectors are represented in the equations (6) y (6).

$$[v_{vi}] = [\dot{q_{pi}}] + [\omega_{Li}] \times (-L_v [e_i]) \tag{6}$$

$$[v_{ci}] = [\omega_{Li}] \times (L_c [e_i]) \tag{7}$$

so $[v_{ci}]$ and L_c correspond to the speed of the center of mass and length of the cylinder y $[v_{vi}]$ and L_v represent the velocity of the center of mass and length of the rod. However, the shape of the equations (6) - (7), is not suitable for dynamic analysis, so it must be expressed in a compact manner, applying the properties of the vector product. Starting with the speed of the rod (6), the equation is replaced (3) in the expression (6), so,

$$[v_{vi}] = [\dot{q_{pi}}] + \frac{[e_i] \times [\dot{q_{pi}}]}{L_i} \times (-L_v [e_i]) \tag{8}$$

applying the anticommutative property,

$$[v_{vi}] = [\dot{q_{pi}}] + (L_v [e_i]) \times \frac{[e_i] \times [\dot{q_{pi}}]}{L_i} \tag{9}$$

then, taking into account the triple cross product present in the equation, the next expression is proposed.

$$[v_{vi}] = [\dot{q_{pi}}] + \left(\frac{L_v}{L_i} \cdot [\dot{q_{pi}}] \right) [e_i] - \left(\frac{L_v}{L_i} \cdot [e_i] \right) [\dot{q_{pi}}] \tag{10}$$

On the other hand, in order to simplify the expression, the term $[e_i]\times$ is $[\hat{e}_i]$, the equation is expressed as,

$$\begin{aligned} [v_{vi}] =& [\dot{q_{pi}}] + (L_v [e_i]) \times \frac{[\hat{e}_i][\dot{q_{pi}}]}{L_i} \\ & [\dot{q_{pi}}] + (L_v [\hat{e}_i]) \frac{[\hat{e}_i][\dot{q_{pi}}]}{L_i} \end{aligned} \tag{11}$$

and finally,

$$[v_{vi}] = \left[[I] + \frac{L_v [\hat{e}_i]^2}{L_i} \right] [\dot{q_{pi}}] \tag{12}$$

In the equation (12) the term $\hat{e}_i$ that is squared, is an anti-symmetric matrix that has the shape of the equation (5) Now, for the cylinder speed, the expression is replaced (3) in the equation (7),

$$[v_{ci}] = \frac{[e_i] \times [\dot{q_{pi}}]}{L_i} \times (L_c [e_i]) \tag{13}$$

Again the properties of the cross product are applied,

$$[v_{ci}] = (-L_c [e_i]) \times \frac{[e_i] \times [\dot{q_{pi}}]}{L_i} \tag{14}$$

$$[v_{ci}] = \left(\frac{L_c [e_i]}{L_i} \cdot [e_i] \right) [\dot{q_{pi}}] - \left(\frac{L_c [e_i]}{L_i} \cdot [\dot{q_{pi}}] \right) [e_i] \tag{15}$$

In the same way that the speed of the rod was done, the expression is simplified,

$$[v_{ci}] = -L_c [e_i] \times \frac{\hat{e}_i [\dot{q_{pi}}]}{L_i} \tag{16}$$

so,

$$[v_{ci}] = \left(\frac{L_c [\hat{e}_i]^T [\hat{e}_i]}{L_i} \right) [\dot{q_{pi}}] \tag{17}$$

Finally,

$$[v_{ci}] = \left[\left(\frac{L_c [e_i]}{L_i} \cdot [e_i] \right) [I] - \left(\frac{L_c [e_i]}{L_i} \cdot [I] \right) [e_i] \right] [\dot{q_{pi}}] \tag{18}$$

Having already the velocity expressions, these are replaced in the equation (2),

$$T_i = \frac{1}{2}\left(\left([I] + \frac{L_v[\hat{e}_i]^2}{L_i}\right)[\dot{q}_{pi}]\right)^T m_v \left([I] + \frac{L_v[\hat{e}_i]^2}{L_i}\right)[\dot{q}_{pi}] + \frac{1}{2}\left(\frac{[\hat{e}_i][\dot{q}_{pi}]}{L_i}\right)^T (I_v + I_c)\left(\frac{[\hat{e}_i][\dot{q}_{pi}]}{L_i}\right) \tag{19}$$

Which can be rewritten in the following way,

$$T_i = \frac{1}{2}[\dot{q}_{pi}]^T \left([I] + \frac{L_v[\hat{e}_i]^2}{L_i}\right)^T m_t \left([I] + \frac{L_v[\hat{e}_i]^2}{L_i}\right)[\dot{q}_{pi}] + \frac{1}{2}[\dot{q}_{pi}]^T \left(\frac{[\hat{e}_i]}{L_i}\right)^T (I_v + I_c)\left(\frac{[\hat{e}_i]}{L_i}\right)[\dot{q}_{pi}] \tag{20}$$

$$T_i = \frac{1}{2}[\dot{q}_{pi}]^T \left(\left([I] + \frac{L_v[\hat{e}_i]^2}{L_i}\right)^T m_t \left([I] + \frac{L_v[\hat{e}_i]^2}{L_i}\right) + \left(\frac{[\hat{e}_i]}{L_i}\right)^T (I_v + I_c)\left(\frac{[\hat{e}_i]}{L_i}\right)\right)[\dot{q}_{pi}] \tag{21}$$

So, the equation (21) can be expressed as,

$$T_i = \frac{1}{2}[\dot{q}_{pi}]^T \left([T_{ai}] + [T_{bi}]\right)[\dot{q}_{pi}] \tag{22}$$

Once the expression of energy is found, it can be partially derived, taking into account that it is a necessary step to reach the Lagrange equation that describes a dynamic system.

$$\frac{\partial T_i}{\partial \dot{q}_{pi}} = \frac{\partial}{\partial \dot{q}_{pi}}\left(\frac{1}{2}\left([\dot{q}_{pi}]^T [T_{ai}][\dot{q}_{pi}]\right)\right) + \frac{\partial}{\partial \dot{q}_{pi}}\left(\frac{1}{2}\left([\dot{q}_{pi}]^T [T_{ci}][\dot{q}_{pi}]\right)\right) \tag{23}$$

$$\frac{\partial T_i}{\partial \dot{q}_{pi}} = (T_{ai} + T_{bi})[\dot{q}_{pi}] \tag{24}$$

Now, the equation is derived (24) with respect to time,

$$\frac{d}{dt}\left(\frac{\partial T_i}{\partial \dot{q}_{pi}}\right) = \frac{d}{dt}(T_{ai} + T_{bi})[\dot{q}_{pi}] + (T_{ai} + T_{bi})[\ddot{q}_{pi}] \tag{25}$$

Finally, deriving the last missing term,

$$\frac{\partial T_i}{\partial [q_{pi}]} = \left(\frac{\partial T_i}{\partial q_{pix}} \frac{\partial T_i}{\partial q_{piy}} \frac{\partial T_i}{\partial q_{piz}}\right)^T \tag{26}$$

Expanding the equation (26)

$$\begin{aligned}\frac{\partial T_i}{\partial [q_{pi}]} = &\left(\frac{m_v L_v}{L_i^2}\right)\left(e_i \dot{q}_{pi}{}^T \dot{q}_{pi} + 2e_i^T \dot{q}_{pi}\dot{q}_{pi} - 3e_i \dot{q}_{pi}{}^T e_i e_i^T \dot{q}_{pi}\right) - \\ &\left(\frac{m_v L_v^2}{L_v^3}\right)\left(e_i \dot{q}_{pi}{}^T \dot{q}_{pi} + \dot{q}_{pi}\dot{q}_{pi}{}^T e_i - 2e_i e_i^T \dot{q}_{pi}\dot{q}_{pi}{}^T e_i\right) - \\ &\left(\frac{I_v + I_c}{L_i^3}\right)\left(e_i^T \dot{q}_{pi}\dot{q}_{pi} + e_i \dot{q}_{pi}{}^T \dot{q}_{pi} - 2e_i \dot{q}_{pi}{}^T e_i e_i^T \dot{q}_{pi}\right)\end{aligned} \tag{27}$$

In this way all the terms of the expression (1) have been obtained, however, as mentioned [16] and it is shown in the Figure 2, this is not the only force present in the actuator, for this reason it is necessary to perform a summation of forces that relates them.

$$\sum F_i^{actuador} = [F_{api}] + m_v[g] + m_c[g] + [F_{pi}] \tag{28}$$

In the equation (28), the term$[F_{pi}]$, is the force vector - existing reactions between the joint of the mobile platform and the actuator, $m_v[g]$ y $m_c[g]$ correspond to the forces present in the actuator due to the mass of the red and the cylinder and $[F_{api}]$ represents the vector of force that makes the movement of the rod. Although the objective of the dynamics is to establish an equation of matrix form, which allows to find the force $[F_{api}]$, first $[F_{pi}]$ is analyzed, in order to replace it in the approach of the equations of the mobile platform and in that way all will be in terms of $[F_{api}]$. Thus,

$$[F_{pi}] = [F_i] - [F_{api}] - m_v[g] - m_c[g] \tag{29}$$

where $[F_i]$ is $\sum F_i^{actuador}$. As each term of $[F_i]$ is known, the equation (29) will be,

$$[F_{pi}] = \frac{d}{dt}(T_{ai} + T_{bi})[\dot{q}_{pi}] + (T_{ai} + T_{bi})[\ddot{q}_{pi}] - \frac{\partial T_i}{\partial [q_{pi}]} - [F_{api}] - m_v[g] - m_c[g] \tag{30}$$

With this the formulation for the dynamics of the actuator is concluded, now the equations for the mobile platform will be considered, according to the diagram shown in the Figure 3

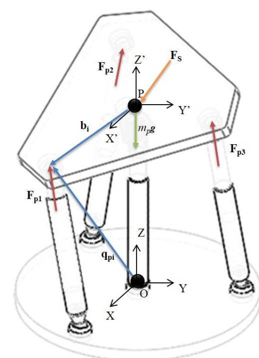


Fig. 3: Position and Force vectors related to the mobile platform

Initially the forces will be analyzed,

$$m_p[\ddot{c}] = -\sum [F_{pi}] + m_p[g] + [F_s] \tag{31}$$

Matrically, this equation is represented by,

$$\begin{bmatrix} m_p[I] & 0 \end{bmatrix}[\ddot{q}] = -\sum [F_{pi}] + m_p[g] + [F_s] \tag{32}$$

To the torques,

$$[I_p][\dot{\omega}_p] + [\omega_p] \times ([I_p][\omega_p]) = -\sum [b_i] \times [F_{pi}] + [d] \times [F_s] \tag{33}$$

However, the equation (33) is referred to the mobile system, it is necessary to work with respect to the fixed system, for which the matrix of transformation $[R]$ is used [17], so,

$$[R]\left([I_p][\dot{\omega}_p] + [\omega_p] \times [I_p][\omega_p]\right) = -\sum ([R][b_i] \times [F_{pi}]) + [R]([d] \times [F_s]) \tag{34}$$

Then, expressing to $[R][\dot{\omega}_p] = [\dot{\omega}]$, a $[R][\omega_p] = [\omega]$, $\omega_p = [\hat{\omega}_p]\times$ y $[R][\hat{\omega}_p][R]^T = [\hat{\omega}]$,

$$[R][I_p][R]^T[\dot{\omega}] + [\hat{\omega}][R][I_p][R]^T[\omega] = -\sum ([R][b_i] \times [F_{pi}]) + [R]([d] \times [F_S]) \tag{35}$$

Now, in the same way as was done with force, the equation(35) must be expressed in a matrix form,,

$$\begin{bmatrix} 0 & [R][I_p][R]^T \end{bmatrix}[\ddot{q}] + \begin{bmatrix} 0 & [\hat{\omega}][R][I_p][R]^T \end{bmatrix}[\dot{q}] = \sum ([R][b_i] \times [F_{pi}]) + [R][\hat{d}][R]^T[F_s] \tag{36}$$

In this way the analysis of the mobile platform is complete, however, it is convenient to situate them in a single matrix,

$$\begin{bmatrix} m_p[I] & 0 \\ 0 & [R][I_p][R]^T \end{bmatrix}[\ddot{q}] + \begin{bmatrix} 0 & 0 \\ 0 & [\hat{\omega}][R][I_p][R]^T \end{bmatrix}[\dot{q}] = \begin{bmatrix} -\sum[F_{pi}] \\ \sum([R][b_i]\times[F_{pi}]) \end{bmatrix} + \begin{bmatrix} m_p[g] \\ 0 \end{bmatrix} + \begin{bmatrix} [F_s] \\ [R][\hat{d}][R]^T[F_s] \end{bmatrix} \tag{37}$$

As seen in the equation (37), the unique unknown term is the forve $[F_{pi}]$, which in turn is in terms of the variable of interest $[F_{api}]$, so that now the expression (30) is replaced in (37), taking into account that there are three actuators,

$$\begin{aligned} &\begin{bmatrix} m_p[I] & 0 \\ 0 & [R][I_p][R]^T \end{bmatrix}[\ddot{q}] + \begin{bmatrix} 0 & 0 \\ 0 & [\hat{\omega}][R][I_p][R]^T \end{bmatrix}[\dot{q}] = \\ &-\sum_{i=1}^{3}\left(\begin{bmatrix} [I] \\ [R][\hat{b}_i][R]^T \end{bmatrix}(T_{ai}+T_{bi})\begin{bmatrix}[I] & [R][\hat{b}_i][R]^T\end{bmatrix}\right)[\ddot{q}] \\ &-\sum_{i=1}^{3}\left(\begin{bmatrix} [I] \\ [R][\hat{b}_i][R]^T \end{bmatrix}(T_{ai}+T_{bi})\,\hat{\omega}^2[R][b_i]\right) - \sum_{i=1}^{3}\left(\begin{bmatrix} [I] \\ [R][\hat{b}_i][R]^T \end{bmatrix}[T_{ci}]\right) \\ &+\sum_{i=1}^{3}\left(\begin{bmatrix} [I] \\ [R][\hat{b}_i][R]^T \end{bmatrix}(m_v[g]+m_c[g])\right) + \begin{bmatrix} m_p[g] \\ 0 \end{bmatrix} \\ &+\sum_{i=1}^{3}\left(\begin{bmatrix} [I] \\ [R][\hat{b}_i][R]^T \end{bmatrix}[F_{api}]\right) + \begin{bmatrix} [F_s] \\ [R][\hat{d}][R]^T[F_s] \end{bmatrix} \end{aligned} \tag{38}$$

Of the equation (38), the term $[\hat{d}]$ is a matrix that have the form of the equation (5) and $[T_{ci}] = \frac{d}{dt}(T_{ai}+T_{bi})[\dot{q_{pi}}] - \frac{\partial T_i}{\partial [q_{pi}]}$ However, the form of the parallel robot dynamic equation is given to [18]:

$$M_{(q)}\ddot{q} + C_{(q,\dot{q})}\dot{q} + G_{(q)} + F_c = H_{(q)}F \tag{39}$$

$M_{(q)}$ is the inertial matrix, $C_{(q,\dot{q})}$ represent the vector of coriolis and centripetal forces, $G_{(q)}$ is the vector of gravitational forces, F_c corresponds to external forces, $H_{(q)}$ is a matrix of unit vectors and F are the internal forces exerted by the actuator. If the terms of the equation (38) are observed and with the expression (39) are compared,is observed that those corresponding to the inertial matrix are $M_{(q)}$,

$$\begin{bmatrix} m_p[I] & 0 \\ 0 & [R][I_p][R]^T \end{bmatrix}$$

$$\sum_{i=1}^{3}\left(\begin{bmatrix} [I] \\ [R][\hat{b}_i][R]^T \end{bmatrix}(T_{ai}+T_{bi})\begin{bmatrix}[I] & [R][\hat{b}_i][R]^T\end{bmatrix}\right)$$

The corresponding to the coriolis and centripetal forces $C_{(q,\dot{q})}$,

$$\begin{bmatrix} 0 & 0 \\ 0 & [\hat{\omega}][R][I_p][R]^T \end{bmatrix}$$

$$\sum_{i=1}^{3}\left(\begin{bmatrix} [I] \\ [R][\hat{b}_i][R]^T \end{bmatrix}(T_{ai}+T_{bi})\,\hat{\omega}^2[R][b_i]\right)$$

$$\sum_{i=1}^{3}\left(\begin{bmatrix} [I] \\ [R][\hat{b}_i][R]^T \end{bmatrix}[T_{ci}]\right)$$

With respect to the term of gravity $G_{(q)}$,

$$\begin{bmatrix} -m_p[g] \\ 0 \end{bmatrix}$$

$$\sum_{i=1}^{3}\left(\begin{bmatrix} [I] \\ [R][\hat{b}_i][R]^T \end{bmatrix}(T_{ai}+T_{bi})\right)$$

The term F_c is composed by,

$$\begin{bmatrix} [F_s] \\ [R][\hat{d}][R]^T[F_s] \end{bmatrix}$$

Finally, for the term of greatest interest is $H_{(q)}F$,

$$\sum_{i=1}^{3}\left(\begin{bmatrix} [I] \\ [R][\hat{b}_i][R]^T \end{bmatrix}[F_i]\right)$$

So, all the terms of the dynamic equation that represents the parallel robot under study have been obtained. To find the forces F_{api}, it is multiplied by the inverse of the matrix H, all the equation.

III. Dynamic Simulation

The simulation is done according to the flow diagram shown in the Figure 4.

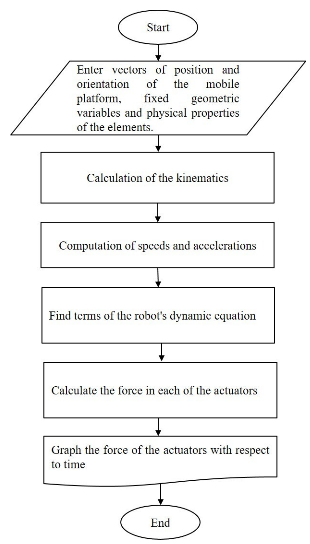


Fig. 4: Dynamic Flow Diagram

It is important to note that the dynamic analysis performed is for a robot with a greater number of degrees of freedom, so that only the geometrical parameters and the physical properties of the robot are adjusted, taking into account what each term corresponds to, according to the explained in the development of dynamic equations. For the simulation the physical properties of the robot were added, such as the mass of the platform, of the rod, of the cylinder $m_p = 4500gm$, $m_v = 630gm$, $m_c = 430gm$, the inertia of the mobile platform $Ixx = 0.034kg\cdot m^2$, $Iyy = 0.067kg\cdot m^2$, $Izz = 0.033kg\cdot m^2$, the inertial products are $Ixy = Iyx = -3.14\times10^{-8}kg\cdot m^2$, $Ixz = Izx = -4.208\times10^{-2}kg\cdot m^2$, $Iyz = Izy = 3.142\times10^{-3}$, the inertias of the cylinder and the rod, $I_c = 1.716\times10^{-3}kg\cdot m^2$, $I_v = 1.658\times10^{-3}kg\cdot m^2$, the lengths of the rod and the cylinder

$L_v = 200mm$, $L_c = 240mm$ and the external force applied on the mobile platform is given by $F_s = [0\ 0\ -600]^T N$. With this data, the program for calculating the dynamics is executed and the following forces are obtained

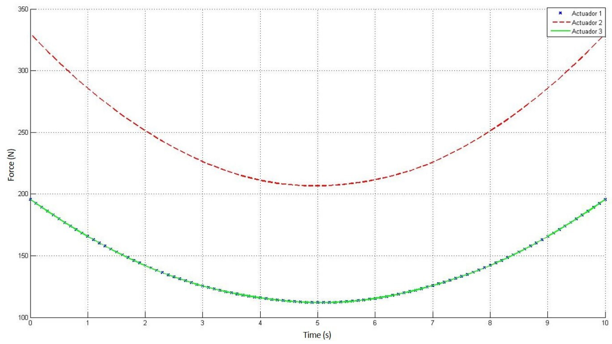


Fig. 5: Dynamic (F_1,F_2, F_3)

According to the Figure 5, the results are the expected ones, since by the symmetry of the structure the actuators 1 y 3, exert the same force for the movement to take place.

IV. Computed Torque Control

If the dynamic model of a system have the form of the equation (40),

$$m\ddot{x} + b\dot{x} + kx = f \tag{40}$$

Where m, b y k are known values and the law of control is organized so that the system is reduced, so that, it seems to have a unitary mass, that is to say, without friction, nor rigidity, with what is posed

$$f = \alpha f' + \beta \tag{41}$$

So α and β are constants o functions that must be chosen in a way that f' is the new entry in the system. So, replacing the equation (41) in (40),

$$m\ddot{x} + b\dot{x} + kx = \alpha f' + \beta \tag{42}$$

According to the expression (42), to obtain the unit mass system with input f', the values of α y β, are select as,

$$\alpha = m$$
$$\beta = b\dot{x} + kx$$

Replacing these expressions in the equation (42),

$$\ddot{x} = f' \tag{43}$$

Now a law of control is proposed,

$$f' = \ddot{x_d} + k_D\dot{e} + k_p e \tag{44}$$

In this e correspond to *error* and it is given to $e = x_d(t) - x(t)$, where $x_d(t)$ is the desired trajectory variant in time. The terms k_D y k_p correspond to the derivative and proportional coefficients respectively. Then replacing the equation (44) in the expression (43),

$$\ddot{x} = \ddot{x_d} + k_D\dot{e} + k_p e \tag{45}$$

Thus,

$$\ddot{e} + k_D\dot{e} + k_p e = 0 \tag{46}$$

Considering that the differential equation obtained is of second order, the step to follow is the choice of the coefficients taking into account that it is sought to obtain critical damping. The diagram that represents the computed torque control is shown in the Figure 6

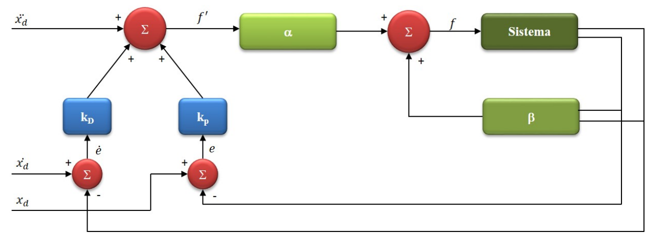


Fig. 6: Block diagram for computed torque control

Now extending the same concept to establish a control law for the parallel platform, tacking account that the characteristic equation that represents the dynamics of the robot is given by the expression (39), so that the output in this case is pose as,

$$[F] = [A][F'] + [\beta] \tag{47}$$

Where $[A]$ represent a matrix and $[\beta]$ a vector defined as,

$$[A] = H^{-1}M$$
$$\beta = H^{-1}C + H^{-1}G + H^{-1}F_c$$

Replacing the terms $[A]$ y $[\beta]$ in the equation (39),

$$[F'] = \ddot{q} \tag{48}$$

In this way the system is represented as one of unitary inertia with an input $[F']$, So a control law can be proposed, so that,

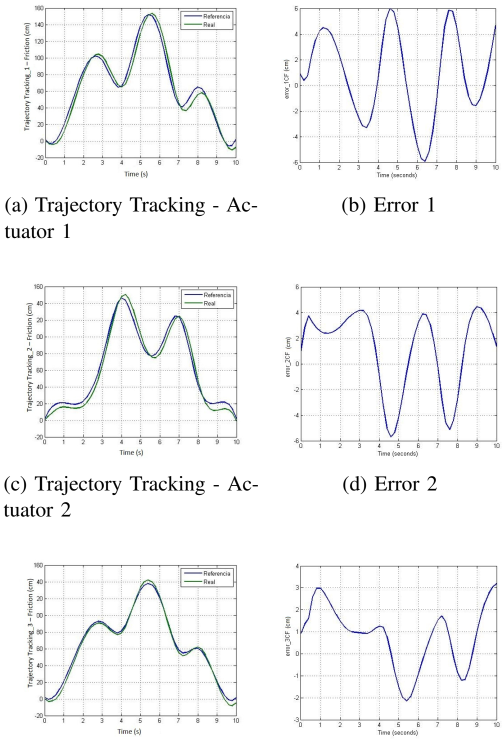
$$[F'] = \ddot{q}_d + [K_D][\dot{e}] + [K_p][e] \tag{49}$$

Where $[e] = [q_d] - [q]$ y $[K_D]$ y $[K_p]$ Represent diagonal matrices of selected gains, with $K_{Dii} = 100$, $K_{Pii} = 50$, with which an overdamped or critically damped performance is achieved, according to the type of trajectory that will be necessary to follow. Finally, the uncoupled error equation is,

$$[\ddot{e}] + [K_D][\dot{e}] + [K_p][e] = 0 \tag{50}$$

V. Simulation Computed Torque Control

For a desired polynomial trajectory where the disturbances, due to the unmodeled friction of the mechanical system, are reflected in each of the actuators, the performance of the controller is shown in the Figure 7



(a) Trajectory Tracking - Actuator 1

(b) Error 1

(c) Trajectory Tracking - Actuator 2

(d) Error 2

(e) Trajectory Tracking - Actuator 3

(f) Error 3

Fig. 7: Controller Performance Computed Torque Control - Trajectories Tracking

Each time the trajectory tracking becomes more complex, the error is greater, however, is important to note that implementing this controller is easier than other techniques and the gains matrices can be adjusted in such a way that performance is improved. Although it is not a robust control technique, it responds well to disturbances.

VI. Conclusion

The dynamic of the parallel robot is very complex, however one of the best methods to analyzed is the Euler - Lagrange Method, compared specially with the Classic Method of Newton Euler [19]. The simulation allowed verify that the mathematical model developed work as expected according to the input reference signals. Once the dynamics is finished, a control law was proposed, that in this case was computed torque control, tacking account that to the serials robots is a strategy very common and with a good performance. In the parallel robot the error was small to the proportion of the robot. To future research other dynamic method can be analyzed to compare the results and the velocity - time of processing, aditionally other techniques of control can be considered, such as sliding mode control or adaptive control, as, these are very good when the system have disturbances. The expressions found for the dynamics are general, that is, these can be used for a robot of n degrees of freedom.

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