A SPINOR BOSON AB CHAIN

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Abstract

We determine the quantum phases of spin-1 bosons loaded in a superlattice type AB. By the Density Matrix Renormalization Group method, we calculate the system energies in the thermodynamic limit in order to get the chemical potential, when the spin-dependent interaction parameter strength and the energy offset is varied. Compared with results of spinless boson on the same superlattice, a significant contribution is obtained in the formation of insulators when the spin is considered, in parameters where these are not energetically favorable for the first case.

Introduction

The spinor Bose gases on potential arrangement have manifested different kinds of quantum phases, such as magnetic order and superfluid, which have become a broad object of study [1, 2]. This kind of gases can be composed by atoms of ^{23}Na, ^{87}Rb, ^{7}Li, ^{41}K, all with total spin $F = 1$, in particular ^{23}Na with antiferromagnetic interaction. Freericks et al. [3] and Rousseau and coworkers [4] found that the Mott insulator of spinor bosons on homogeneous lattices, considering ferromagnetic and antiferromagnetic interaction, presents a decreasing critical point, when the exchange parameter U_2 takes increasing values and the Mott lobes are smaller compared with the case of spinless bosons. Besides, when non-integer densities are considered, always a superfluid phase is present. Warner et al. [5] did a study about spinless and spin-1 bosons in superlattices. Using mean-field approximation, they presented results of spin-1 bosons with the parameters $U_2/U_0 = 0, 04$, being U_2/U_0 the ratio between the spin-dependent parameter over the on-site repulsion interaction. Larger insulator areas were gotten when it was considered spin-dependent and homogeneous lattice. The latter is due to the presence of even number of particles, which favours the creation of singlets.

Model

A spin-1 gas in a superlattice can be described with the Bose-Hubbard model with two additional terms: the on-site spinor interaction and the energy shift at each unit cell given by

$$\widehat{H} = -t \sum_{<i,j>,\sigma} (\widehat{a}^\dagger_{i,\sigma}\widehat{a}_{j\sigma} + \widehat{a}^\dagger_{j,\sigma}\widehat{a}_{i\sigma}) + \frac{U_0}{2}\sum_i \widehat{n}_i(\widehat{n}_i - 1) + \frac{U_2}{2}\sum_i(\widehat{S}_i^2 - 2\widehat{n}_i) + \sum_i \lambda_i \widehat{n}_i, \quad (1)$$

where t is the hopping parameter to nearest-neighbour, $\widehat{a}^\dagger_{i,\sigma}(\widehat{a}_{i,\sigma})$ is the creation (annihilation) operator of a boson at site i in the magnetic sublevel $\sigma = 1,\ 0,\ -1$, $\widehat{n}_i$ is the number operator, U_0 describes the repulsion on-site interaction between bosons while U_2 is the exchange interaction parameter, the spin operator is given by $\widehat{S}_i$ and λ_i is the parameter that characterizes the potential of the superlattice. Besides $\frac{U_2}{U_0}$ satisfies:

$$-1 < \frac{U_2}{U_0} < \frac{1}{2}. \quad (2)$$

Atomic Limit

When $t = 0$ is considered, it is possible to realise the contribution of the antiferromagnetic interaction ($\frac{U_2}{U_0} > 0$). If ΔE denotes the change in the energy when one particle is removed of the system, we obtain a series of boundary lines which divides the phases in the phase diagram. In Fig. 1 we observe two cases when $\frac{U_2}{U_0} = 0$ (spinless bosons) and $\frac{U_2}{U_0} \neq 0$ (spin-1 bosons).

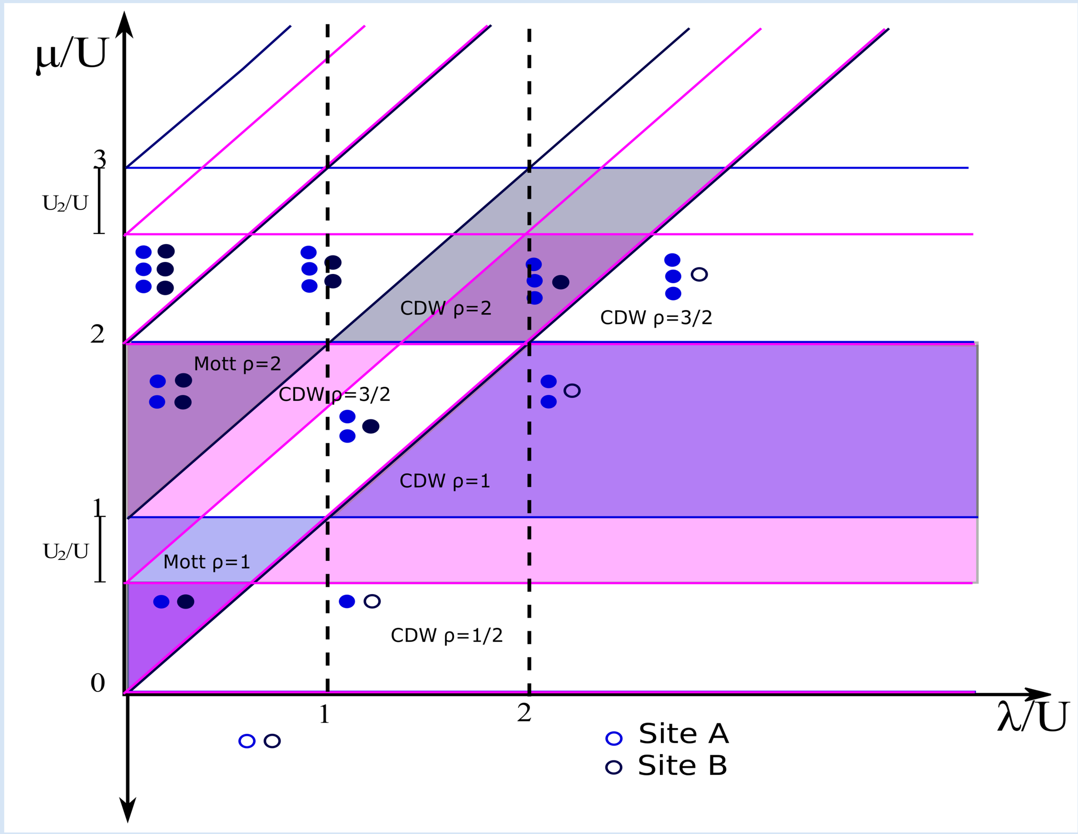

Fig. 1: Atomic limit for spinless (blue lines) and spin-1 bosons (pink lines). We have two types of lines: horizontal which denotes the boundaries phase in the site A ($\frac{\mu}{U_0} = n_A - \frac{U_2}{U_0}$) and diagonal lines which denotes the boundaries in the sites B ($\frac{\mu}{U_0} = \frac{\lambda}{U_0} + n_B - \frac{U_2}{U_0}$). We can observe how the pink regions are deformed while U_2 increases. For example, in $\lambda/U = 1$ it is shown the rising insulator phases.

Results

For the AB chain, Fig 2:

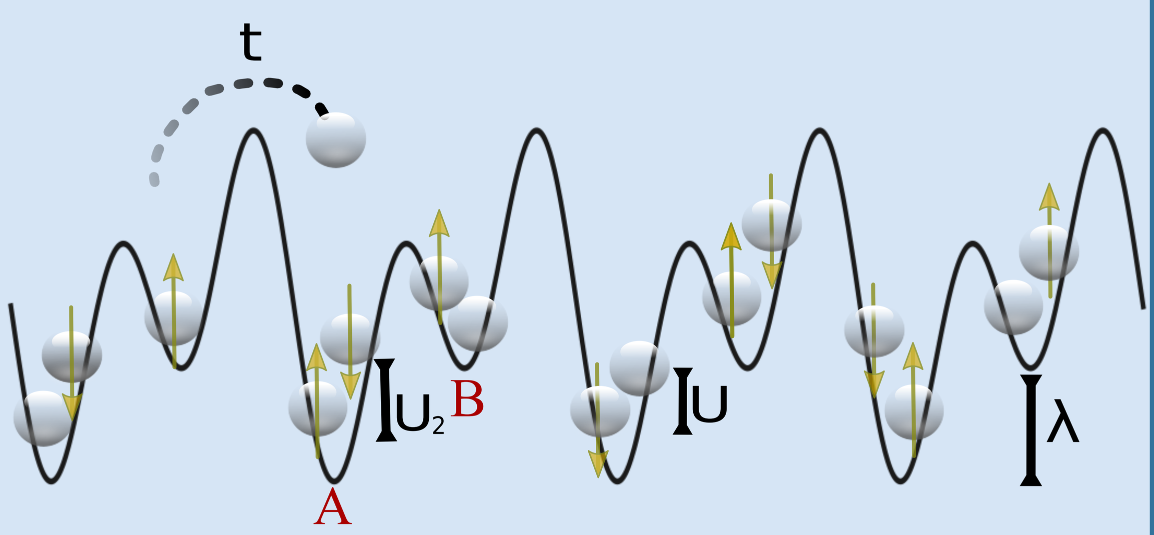

Fig. 2: Spin-1 bosons on AB chain.

we obtain new density profiles, as is shown for $\lambda/U_0 = 9.5$ comparing $U_2/U_0 = 0$ and $U_2/U_0 = 2$ in Fig. 3.

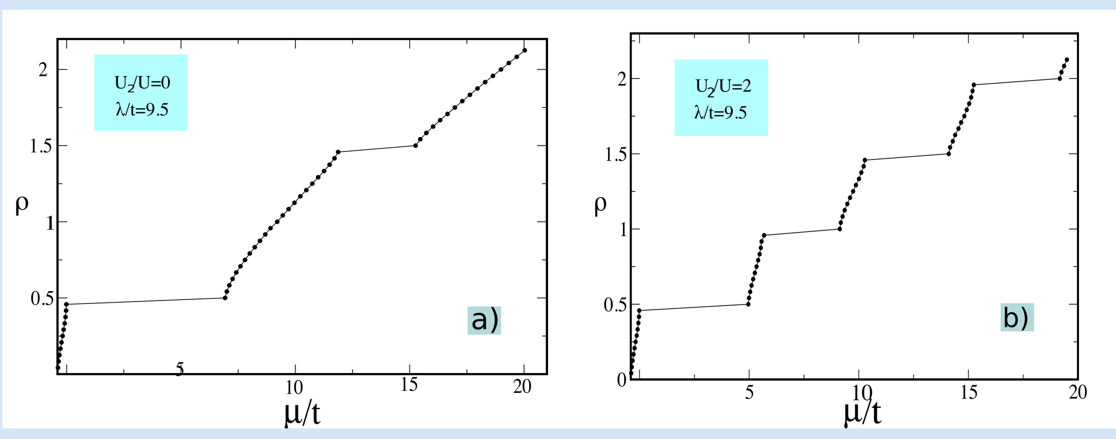

Fig. 3: Density profiles for bosons on AB chain. a) Profile for spinless boson in $\lambda/U = 9.5$, we can see plateaus of insulator with semi-integer densities. b) The plateaus in insulators with $\rho = 1, 2$ appear in values of $U_2/U \neq 0$.

We present the phase diagram for a particular value of λ, and the behaviour of μ against U_2 in Fig. 4:

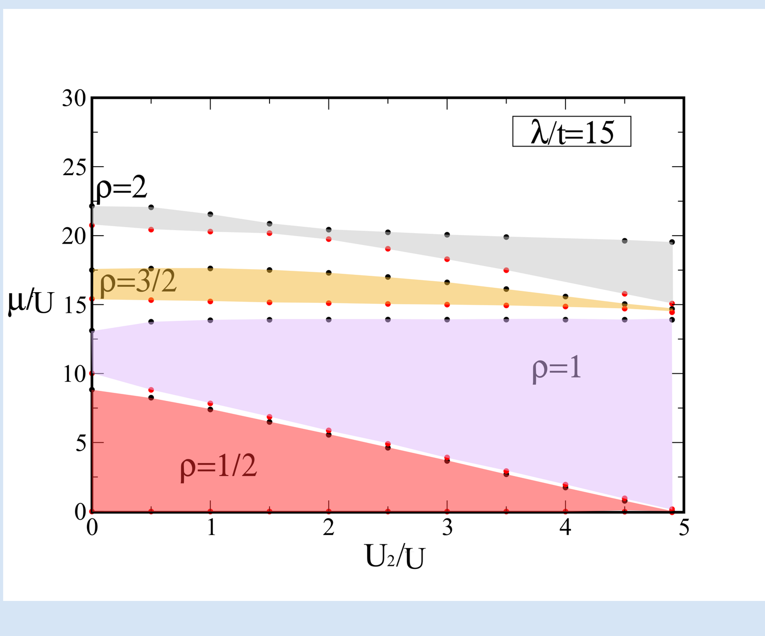

Fig. 4: Phase diagram μ vs U_2 for $\lambda/t = 15$.

Phase Diagram of Spin-1 Bosons

By Density Matrix Renormalizaton Group we calculate the phase diagram of spin-1 bosons in $\lambda/t = 9.5$, as is shown in Fig. 5.

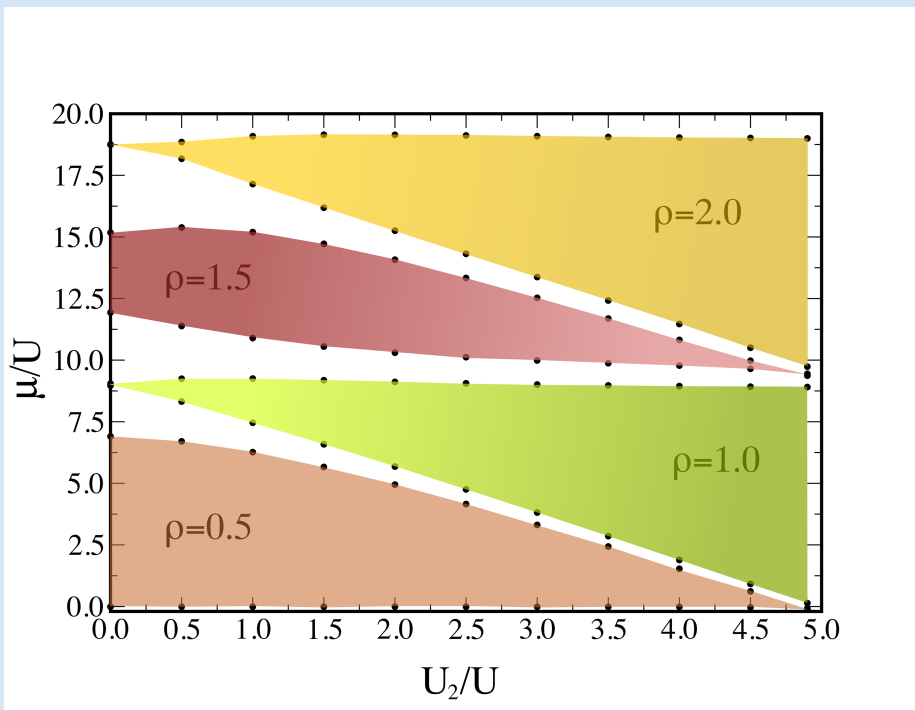

Fig. 5: Phase diagram μ vs U_2 in the thermodynamic limi for $\lambda/t = 9.5$.

For this case, we can observe that areas of CDW phases in semi-integer densities decrease when the spin-dependent interaction increases, and with the analysis of the atomic limit we can see that the insulator with density $\rho = 1$ is CDW phase, and the insulator in $\rho = 2$ is Mott insulator.

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