A New method to measure slope in large rivers using tracers.

A.J. Constain*, D.J. Mesa**, C.A. Peña-Guzmán**, J.L. Corredor***

*Fluvia Tech, Bogotá, Colombia,(E-mail: alfredoconstain@gmail.co)
**Universidad Santo Tomas, Bogotá, Colombia, (E-mail: duvanmesa@usantotomas.edu.co, carlospena@usantotomas.edu.co)
***Universidad Militar Nueva Granada, Bogotá, Colombia. (Email: jorge.corredor@unimilitar.edu.co)

Abstract.
The use of tracers on small and medium rivers is a practice very common for calculate hydraulics characteristics and its hydrodynamic with excellent results, however in large rivers has not been common the use tracers for calculate hydraulics variables. For that reason some researchers used the concept about transport coefficient as time function to calculate the slope of large rivers by means of Elder´s equation presented in 1959. The aim of this paper is to show why this equation is quite correct, despite the incongruence with these “reference values” which had an inaccurate foundation. Put in this new perspective, the Elder´s equation may give useful values for slope, especially in those cases where is very difficult to measure by means of other methods. It is presented here a detailed experimental application of methodology to a large river in Colombia with great results.

Key words
Fluvial measurements, geomorphology, large river, tracers.

INTRODUCTION

The slope as an important parameter in Chezy-Manning´s and Elder´s equations

One of the most important characteristics when describing a flow is the slope, *S*, especially in the study of streams since it is related to other geomorphologic parameters with the well-known Manning-Chezy equations (1) and (2), where *U* is the mean velocity in uniform flow, *C* the Chezy coefficient, *n* the surface roughness number and *R* the hydraulic radius.

$$U = C\sqrt{RS} \tag{1}$$

$$C = \frac{R^{\frac{1}{6}}}{n} \tag{2}$$

Currently in practical field applications of these equations, slope is stablished as an *external* parameter which should be measured approximately by topographic means. Additionally there is an analytic approach by means of the Elder´s equation (Elder, 1959) which links depth, *h*, slope, *S*, the longitudinal dispersion coefficient, *E*, and the gravity acceleration, $g \approx 9.81\ m/s^2$.

$$E = 5.93\, h\sqrt{h\ g\ S} \tag{3}$$

J.W. Elder proposed the equation (3) after several successful laboratory experiments in which he could describe the dispersion transport of a solute in a small flume device. To do that he related the shear effects on mixing process in turbulent flow, also he used the vertical velocity distribution in a flow to get the pattern of impulse transference among small parcels of fluid.

Some problems to extent Elder´s equation to natural streams

Despite these initial successes, it was very difficult to apply the equation to larger natural streams. In these cases, the obtained data for *E* were smaller than those calculated as "reference values" by means of the "Routing procedure", a method considered in that time as the ideal one. It was based on correlation of experimental curve with an ideal Gaussian model and was proposed by H.B. Fischer (Fischer, 1967).

This lack of coincidence of Elder´s results compared with the Fischer´s reference values was explained in terms that actually the vertical distribution of velocities did not describe accurately the causes of shear random separation mechanisms for solute particles in flow; instead the transverse distribution should be used in order to represent them in a better way. Because of this, the usage of Elder´s equation was seriously limited in several applications.

A modern review of Elder´s equation

Due to the importance of Elder´s equation itself, in this paper will be reviewed the objections made on equation (3) and also put the parameter *E*, as a more general fashion than currently, especially as a time function. In this way it is possible to recover this equation to apply it in natural streams. Then, this new procedure may be applied to calculate slope in larger rivers getting a set of congruent and useful information.

Validity of "Routing procedure" as a reference method to calculate E. The so called reference method to find a proper value for the longitudinal dispersion coefficient, *E*, considered as "ideal" due to its foundation on a correlation process of experimental curve on a Gaussian curve, had also a very serious drawback because experimental shape is *always* remarked non-Fickian while Gaussian curve is *always* almost symmetrical, then there is an essential problem with this approach. This fact observed only some years later (McQuivey & Keefer, 1974) these led to consider "Routing procedure" as an also a non-accurate one in following decades. So if this "reference" does not give accurate values on E, it is not a fair judgment mean to other specific methods (among them, the Elder´s)

Equivalence of vertical and transverse distribution of velocities in a turbulent flow. The remaining objection to accuracy of Elder´s equation lays on the fact that, apparently there is an essential difference between vertical and transverse distributions of velocities in turbulent flows, leading to an error when random shear transport is examined as function of one or another distribution. This assertion is often stated because in natural streams, currently the wide is greater than depth, and variations on velocity are more evident in transverse sense than in vertical sense. This view *would be* right if transverse and vertical random motions would be statistical *independent*, in which case a linear exact differential equation would be stated as, where v_z and v_y are the vertical and transverse distribution deviations of velocity:

$$dE = \frac{\partial E}{\partial v_y} dv_y + \frac{\partial E}{\partial v_z} dv_z = dE(v_y) + dE(v_z) \tag{4}$$

In this case, d$E(v_z)$ and d$E(v_y)$ are in competence one against the other, and it is wrong does not consider one or other. But if by the other hand, there is statistical *dependence* between v_y and v_z, it is correct to write:

$$dE = k_y dE(v_y) \approx k_z dE(v_z) \tag{5}$$

In this case one or other function is acceptable to define dE (they are not in competence). Coefficients k_y and k_z simply measure the relative importance of each kind of deviation distribution. So it is right to put Elder´s equation as function of one or other distribution. If the flow turbulence is considered isotropic and uniform (as usually) then the dependence between distributions v_y and v_z is assured.

The longitudinal dispersion coefficient as function of time

But it is not the equivalence of transverse and vertical distribution of velocities the only feature that gives a new power to Elder´s equation. The time dependent nature of the longitudinal transport coefficient is of most importance to understand the new applications of this equation to measure large rivers.

The complete application of Galileo´s transformation to describe the solute plume. It is a well-known the Ficks´equation (6) as solution to Taylor´s basic differential mass balance in flows (7). Here C_s is the solute concentration in flow and A is the cross section area of flow. M is the solute of mass poured to flow suddenly.

$$C_s = \frac{M}{A\sqrt{4\pi\, E\, t}} e^{-\frac{(X - U\, t)^2}{4E\, t}} \tag{6}$$

$$\frac{\partial C_s}{\partial t} = U \frac{\partial C_s}{\partial x} + E \frac{\partial^2 C_s}{\partial X^2} \tag{7}$$

Although almost never is said when is discussed this equation (6), the Galilean transformation should be applied to describe properly the solute dynamics from one specific point of view, the “observer frame” in classical mechanics. In the current case of this description, the observer is fixed in border and he (or she) sees the plume pass on with the mean velocity U, and equation (6) is considered as the correct one. If a second observer is considered, this time moving over the peak of plume, he (or she) must use another kinematic description, with a distinct frame, *X´:*

$$C_s = \frac{M}{A\sqrt{4\pi\, E\, t}} e^{-\frac{(X')^2}{4E\, t}} \tag{8}$$

This is obvious if one think that this second observer travels with U also and he (or she) eliminates this velocity for the plume kinematics. Until here all seems correct, as matter of fact current description in books is like that. However, it is not difficult to show that additionally *E* must be not a constant (as are taken usually) but *a time function*. Only in this way, equations (6) and (8) may describe properly both observers´ description of solute plume in flow, as mechanical relativity principle requires.

The Elder´s equation using *E* as time function.

If the before definition of *E* as time function is true, then the Elder´s equation must be used in a specific point which may be or may not be coincident with the properly estimated value for *E(t)*. This is a main reason why "reference" methods" (as for example "Routing procedure") often did not give proper values for *E*, when used whit this equation, additionally to the Non-Fickian distortion discussed before.

If this feature is correct, a method to find the value of slope may be devised, especially for larger rivers.

Finding the proper value of *E(t)* for Elder´s equation.

When one is describing the motion of a solute plume, there are several parameters to observe. First, the advection time, *t* is related with motion of plume as a whole. The characteristic time, τ is a measure of the motion of dispersive particles, related with *t* applying the Poisson´s distribution.

$$\tau \approx 0.215 \times t \tag{9}$$

Second, there is a state function, $\boldsymbol{\phi}$, which relates the mean dispersion velocity of solute particles in a shear process with this advection velocity. The corresponding characteristic displacement, Δ of solute particles is one dimension Brownian in nature.

$$\phi = \frac{V_{disp}}{U} = \frac{\left(\frac{\Delta}{\tau}\right)}{U} = \frac{\left(\frac{\sqrt{2E\tau}}{\tau}\right)}{U} \tag{10}$$

Then, equaling the Chezy classical result with the new mean velocity definition from equation (10), it states as follow:

$$C\sqrt{RS} \approx \frac{1}{\phi}\sqrt{\frac{2E}{\tau}} \tag{11}$$

And then:

$$S \approx \frac{E^2}{35.2 \times h^3 \times g} \tag{12}$$

Using equations (3) and (11) we finally have an "estimation function", *F*, as follows:

$$F \approx \phi^2 \tau \left(\frac{C^2}{2}\right)\left(\frac{R}{h}\right)\sqrt{\frac{S}{hg}} \tag{13}$$

If $F \approx 5.93$ the slope is probably a good value for the considered stream.

METHODS

For evaluate the method to find the slope described here, was applied an experimental test in Meta river located in eastern Colombia. The Meta River rises in the Eastern Cordillera and has an approximate length of 804 km the width is between 600 and 2100 m and with 3200 m^3/s of flow and a mean depth of 4.1 m, estimated in the stretch of the river (Using data of The Institute of Hydrology, Meteorology and Environmental Studies in Colombia (IDEAM.) Figure 1

Figure 1 A view of Meta River in the section of measurement

For this study it was done four pouring, two in each border of the River, one border was named "Casanare" and "Casanare 1" and the other "Vichada" and "Vichada 1" as shown in the following map. Figure 2.

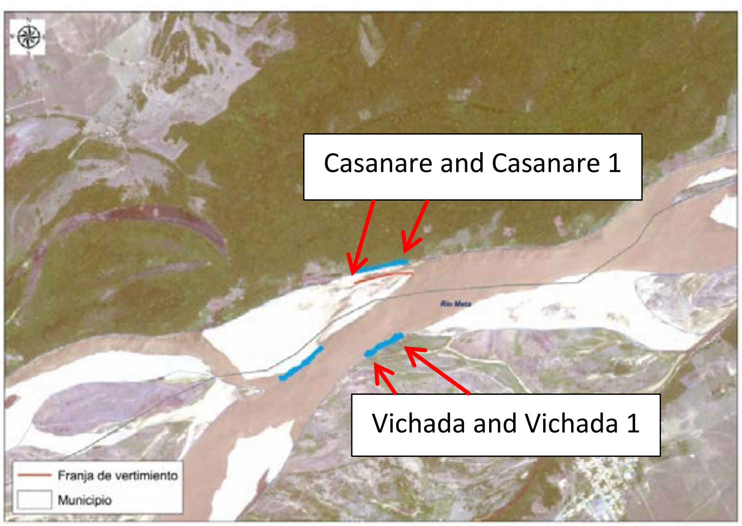

Figure 2. Map of the measured trench in Meta River (blue lines).

The experimental work started with the injecting fluorescent tracer near the borders to know the ϕ function that reflects the thermodynamics of the entire section of the stream (in the same sense that the conductivity background of all measured section of the stream is essentially the same). This is a well-known characteristic of a stream near dynamic equilibrium in which the maximum entropy principle lead to a uniformity (equal probability) of some general parameters. (Leopold L & Maddock T., 1953)

Next step was calculated the longitudinal dispersion coefficient since it is the main theoretical tool that allows us to know the slope by means of Elder´s equation developed in time. Once calculated the coefficient following is to evaluate the mean velocity of stream to put it into equation (14).

$$E(t) = \frac{\phi^2 U^2 \times 0.215 \times t}{2} \tag{14}$$

The velocity in Meta River follows approximately a typical distribution as in Figure 2, being near zero in borders and with a maximum value in the center of stream. This characterization is very important because it is too difficult to face with the very complex hydrodynamic of this very large river without a theoretical view. With this approximate tool it is possible to estimate the ratio of velocity of flow, U, to maximum velocity (in the center), U_{max}, as function of ratio of local wide, Wl, to center wide W, Wl/W. Also it is possible to calculate the mean value of this scheme that is as follows:

$$U_m \approx 0.92 \times U_{\max} \tag{15}$$

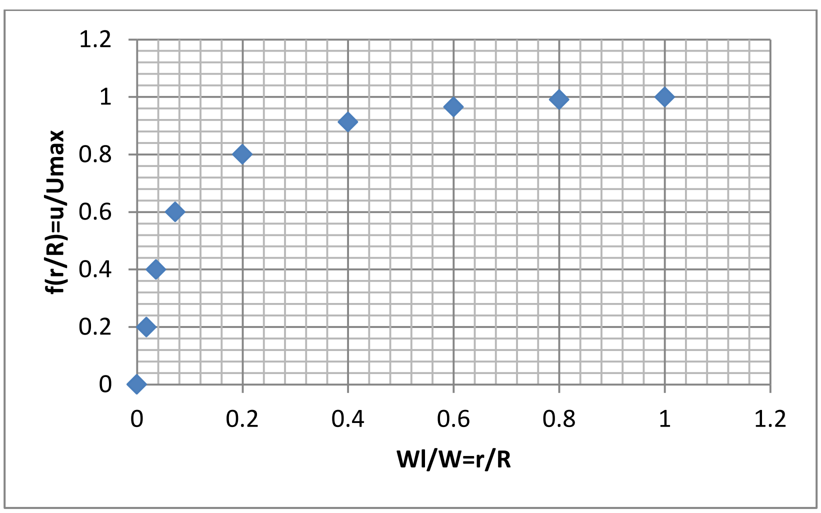

Figure 2 Normalized transverse distribution of velocities from a border (0) to center (1) of the river

RESULTS AND DISCUSSION

By means of tracer was measured approximately the maximum stream velocity in the center of stream, using GPS and chronometer. This value was $U_{max} \approx 2.1\ m/s$ then using equation (15) to know the mean velocity assigned to whole section of stream.

$$U_m \approx 0.92 \times 2.1 \approx 1.93\ m/s \tag{16}$$

With this value it is possible to calculate the time elapsed in the tracer transport in every pouring put in terms of center dynamic.

$$t_j\,(\text{for each pouring "}j\text{"}) \approx \frac{X_j}{1.93} \tag{17}$$

Once completed the tracer study were calculated: mean velocity, effective transport time, state function and longitudinal dispersion coefficient, as show in the Table 1.

Table 1. Experimental data for 4 pouring in River Meta

Pouring	Mean velocity U_m(m/s)	Effective transport time. t (s)	State function. Φ	Longitudinal dispersion coefficient E (m^2/s)
Casanare X=37 m M=52 g	1.93	19.4	0.70	3.77
Casanare 1 X=200 m M=50 g	1.93	66.4	0.28	3.27
Vichada X=43m M=100 g	1.93	430	0.33	0.97
Vichada 1 X=100m M=80 g	1.93	400	0.156	0.506

The values of longitudinal dispersion coefficients are calculated in a very congruent way, regarding a “whole stream trench” thermodynamic parameter, $\boldsymbol{\phi}$, and mean velocity. Then, these values are a good characterization of transport in this section of the stream.

Calculation of slope

With before presented data it is possible to obtain now the slope values for the 4 pouring points, using equation (12) and the estimation function *F*, as show in the Table 2.

Table 2. Slope values for 4 pouring in River Meta

Pouring	E (m^2/s)	S	F
Casanare	3.77	0.00037	5.936
Casanare 1	3.27	0.00028	5.903

Vichada	0.97	0.00003	5.906
Vichada 1	0.506(*)	0.000007 (*)	5.680
Average	2.67	0.00023	5.920

(*) It is a gross mistake and it is not taken into account.

A correspondent Manning number is near n≈0.022 which is close to expecting value for this kind of stream. This appreciation is supported by two independent sources: The values are in good congruence with those obtained by application of Leopold-Maddock model to this trench, and the appreciation of fluvial hydraulics specialist who was informed about the nature and localization of Meta River according with Map of Figure 2. i.e: a “plain river” not so close to mouth in sea.

CONCLUSIONS

1.-It is important to remark that the simplicity and easy application of the new method may be very useful in certain cases where data is too difficult to obtain by other methods.

2. - This new method may help to known congruent values in geomorphology applied to water quality studies. This is a very significant issue regarding that up to date current models (as for example the well-known QUAL2K) requires statistical “homogenous" values to give accurate results.

3. - The values find to Meta River was close to those expected according current methodology, i.e. Observations of specialist regarding roughness characteristics of this type of flows, and values calculated by means of Leopold-Maddock equations.

4. - Obtained results in Meta River in Colombia validate the se of RWT tracer as useful in measuring tasks in large rivers, if transport coefficient is a time function applied to Elder´s equation.

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